CONTINUING COMMENTS ON "THE UNCERTAINTY IN ACCIDENT CONSEQUENCES CALCULATED BY LARGE CODES DUE TO UNCERTAINTIES IN INPUT"

One of the principal points of our comments[1] on the paper by Nguyen[2] was that the method of moment matching to a member of a prescribed family of distributions is potentially unreliable. Neither the author's reply[3] nor the further comments by Baker[4] addressed this issue.

As we pointed out,[1] a probability distribution need not be determined even by its entire infinite sequence of moments. Figure 4 of Ref. 1 shows two very different distributions with a common set of moments. The matching of distributions based on only the first *four* moments is, *a fortiori*, suspect, and may introduce large errors. The size of such errors may be estimated based on formulas given in Ref. 5.

Consider, for example, a distribution whose first four moments are those of the standard normal distribution, i.e., $M_1 = M_3 = 0, M_2 = 1, M_4 = 3$. Sharp upper and lower bounds on the cumulative distribution function are as follows[5]:

$$F_U(x) = \frac{2}{x^4+3} \ , \qquad \text{if} \quad x < -1$$

$$= \frac{2}{x^4+3} + \frac{[1 + x(x^4 - 3x^2 + 3)^{1/2}](1 - x^2)}{2(x^4 - 3x^2 + 3)^{1/2}[(x^4 - 3x^2 + 3)^{1/2} - x^3]} \ , \qquad \text{if} \ -1 < x < 1$$

$$= 1 \ , \qquad \text{if} \quad x > 1$$

$$F_L(x) = 0 \ , \qquad \text{if} \quad x < -1$$

$$= \frac{[1 + x(x^4 - 3x^2 + 3)^{1/2}](1 - x^2)}{2(x^4 - 3x^2 + 3)^{1/2}[(x^4 - 3x^2 + 3)^{1/2} - x^3]} \ , \qquad \text{if} \ -1 < x < 1$$

$$= \frac{x^4+1}{x^4+3} \ , \qquad \text{if} \quad x > 1 \ .$$

These bounding curves are shown in Fig. 1. It follows that the 97.5th percentile of the distribution may be any number between 0.43 and 2.96. Thus, the indeterminacy in the 97.5th percentile, which is the upper bound of a 95% confidence interval, is of the same order of magnitude as the length of that interval for a standard normal variable. In other words, matching of distributions based on the first four moments may introduce uncontrollable errors of the same order of magnitude as calculated quantities of interest.

We previously recommended[1] the use of the Monte Carlo method in this type of work. Its crucial advantage over the moment matching method lies in the feasibility of error estimation. The true distribution and the distribution obtained by Monte Carlo simulation differ only because of sampling error, the size of which can be readily estimated by standard methods.

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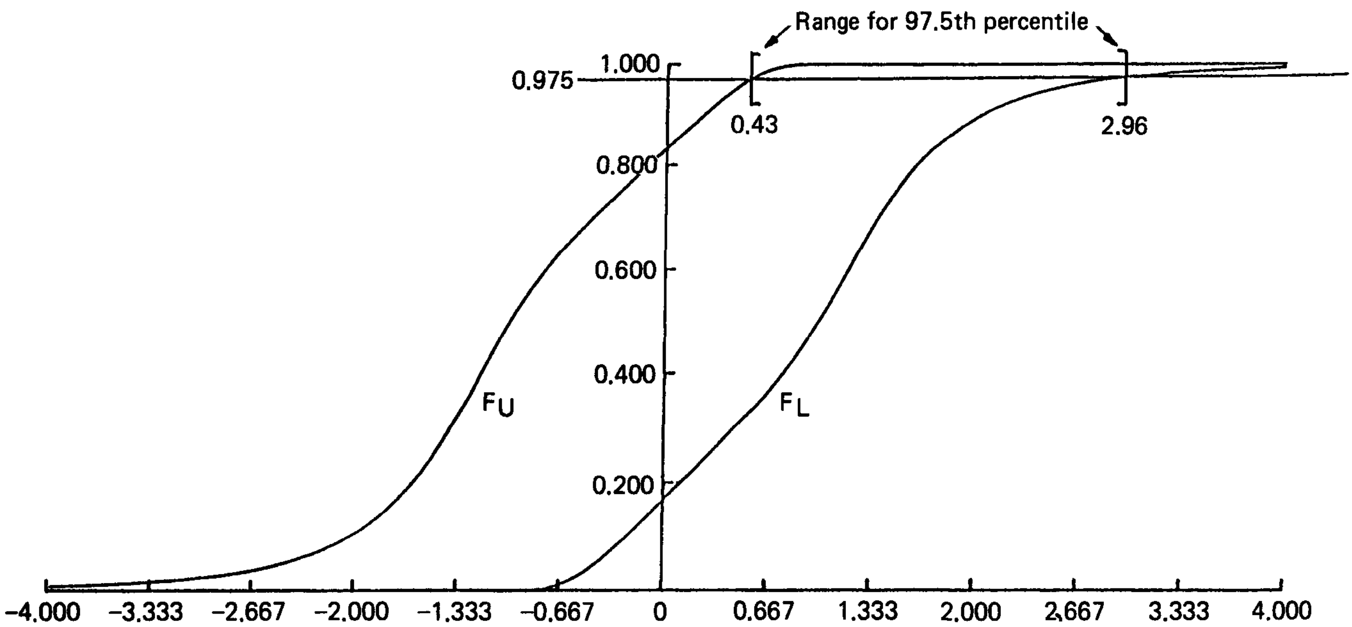

Fig. 1. Sharp upper and lower bounds for a cumulative distribution function with moments $M_1 = M_3 = 0, M_2 = 1, M_4 = 3$.

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REPLY TO "CONTINUING COMMENTS ON 'THE UNCERTAINTY IN ACCIDENT CONSEQUENCES CALCULATED BY LARGE CODES DUE TO UNCERTAINTIES IN INPUT' "

Cox et al.[1] have had the opportunity to make their point that the Monte Carlo method is preferable over the moment matching method for the construction of probability distribution functions. Their continued use of my paper[2] to press the issue indicates that they have missed the point in my reply[3] to their earlier critique.[1]

The objective of my work[2] was to determine the confidence level in the calculation of fuel pin failure time using an accident code. For this particular application, I was satisfied that the moment matching method was acceptable: the shapes of the distributions obtained by this method[3] did not differ significantly with those obtained by the Monte Carlo method.[1] It was not an objective of my work[2] to determine the potential error of the moment matching method when applied to other specific cases. Furthermore, I have not applied the Monte Carlo method to the same problem, and therefore cannot evaluate the relative accuracy of the two methods for this problem.

Such an evaluation does exist, however.[4] Reference 4 compared the Monte Carlo method and the moment matching method and concluded that "the two methods of uncertainty analysis produce densities with no significant difference." Reference 4 went further to recommend the Response Surface Method (which used the moment matching technique) as the more economical method of uncertainty analysis.

I believe that few would question the potential accuracy which the Monte Carlo method could provide. But even with today's computer speed, this accuracy is being paid for at a high price. In the uncertainty analysis of large computer codes, economy has become an overriding consideration. Furthermore, one cannot deny the fact that the moment matching method can be and has been successful, as part of an economical tool for uncertainty analysis.[4] It appears to me that there is a trade-off betwen the two methods, and it would be more constructive to define criteria foı ıhis trade-off than a generalized condemnation of a particular method based on specific examples. Such trade-off criteria should account for not only accuracy, but also economy and convenience in connection with the uncertainty analysis of large codes.

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