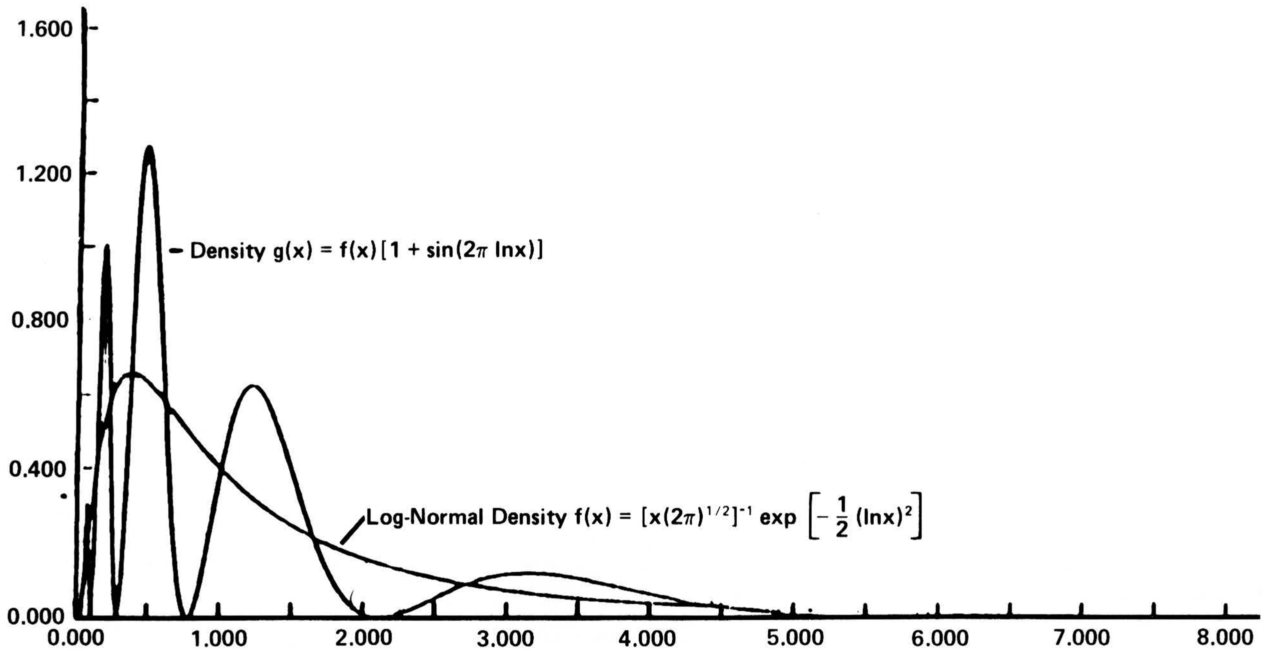

Fig. 4. Two different pdf's with the same moments.

obtain confidence bounds for any quantity of interest, and uncontrollable sources of error are thereby avoided.

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July 11, 1980

REFERENCE

1. DONG H. NGUYEN, "The Uncertainty in Accident Consequences Calculated by Large Codes due to Uncertainties in Input," *Nucl. Technol.*, **49**, 80 (1980).

REPLY TO "COMMENTS ON 'THE UNCERTAINTY IN ACCIDENT CONSEQUENCES CALCULATED BY LARGE CODES DUE TO UNCERTAINTIES IN INPUT' "

The critique of Cox et al.[1] of my paper[2] can be summarized as follows:

1. There are discrepancies between Fig. 3 and Eqs. (14) and (15).
2. The probability density functions (pdf's) of the calculated output for the two cases considered in my paper (inputs with all uniform distributions and inputs with mixed uniform-normal distributions) should not differ significantly. The source of error was speculated to originate from the methods of error propagation, followed by moments matching to a member of a prescribed family of distributions.
3. In general, "the output distribution should not be particularly sensitive to the precise shapes of the input distributions."

Concerning the first point, it is recalled that in my work, the second-order error propagation was calculated by the program SOERP (Ref. 3), yielding the mean, the variance, and the coefficients of skewness and kurtosis. These properties were then used in the program PDFPLOT (Ref. 4) to match a theoretical distribution or a member of an empirical family of distributions. The PDFPLOT performed intermediate calculations using standardized random variable (mean = 0, variance = 1), then rescaled back to the actual random variable for plotting. The discrepancies between Fig. 3 and Eqs. (14) and (15) in Ref. 2 are caused by the fact that the constants given following these equations are for the standardized, not for the actual, variable, while Fig. 3 shows unnormalized distributions of the actual variable. Also, the normalization constant should be $K = 1/1.369$. For the given sets of inputs, PDFPLOT calculated the correct distributions, the discrepancies being caused by an inconsistency on my part. But, this is not a very important problem.

The major critique is in the shapes of the distributions shown in Fig. 3 of Ref. 2. The basic argument that the output distributions resulting from two sets of inputs, each having symmetric unimodal, although different, distributions should not differ significantly appears plausible. Therefore, the details of the calculations were carefully reviewed and an error was found in the inputs to SOERP for the moments of the standardized rectangular distribution. The central moments μ_r of the distribution

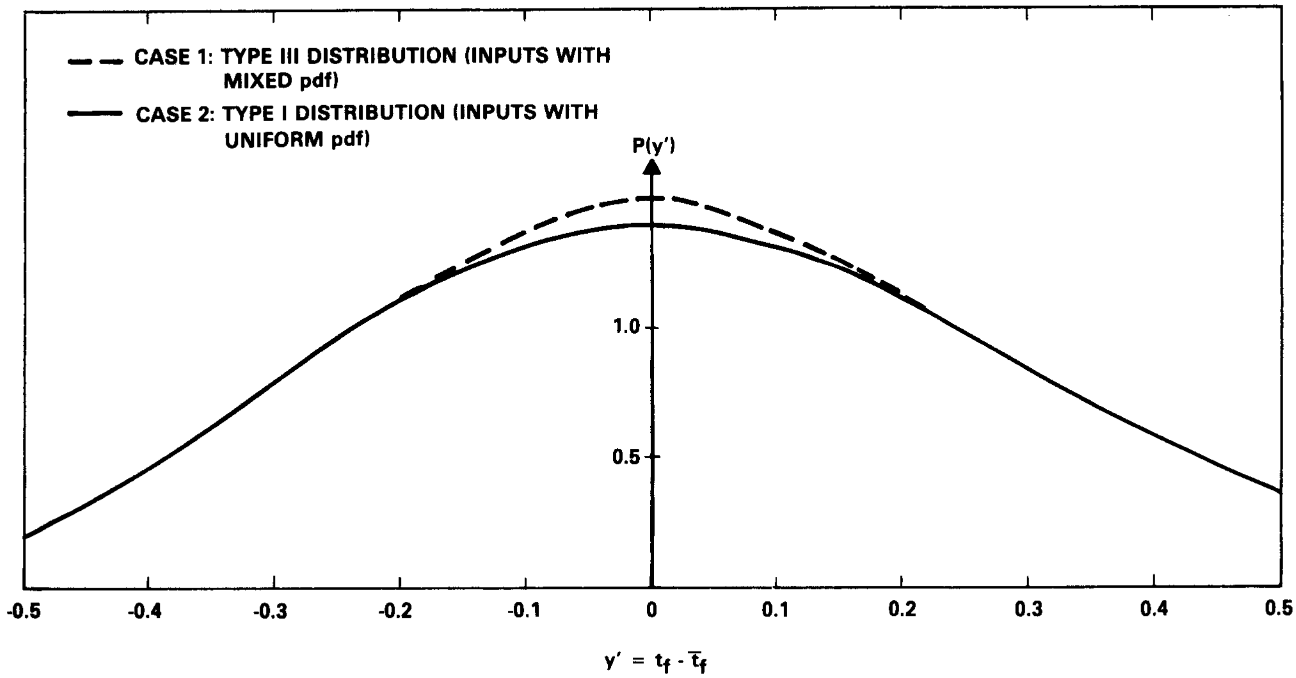

Fig. 1. The pdf of time-of-failure for different combinations of input distributions (σ_i = 5 to 10% of nominal values).

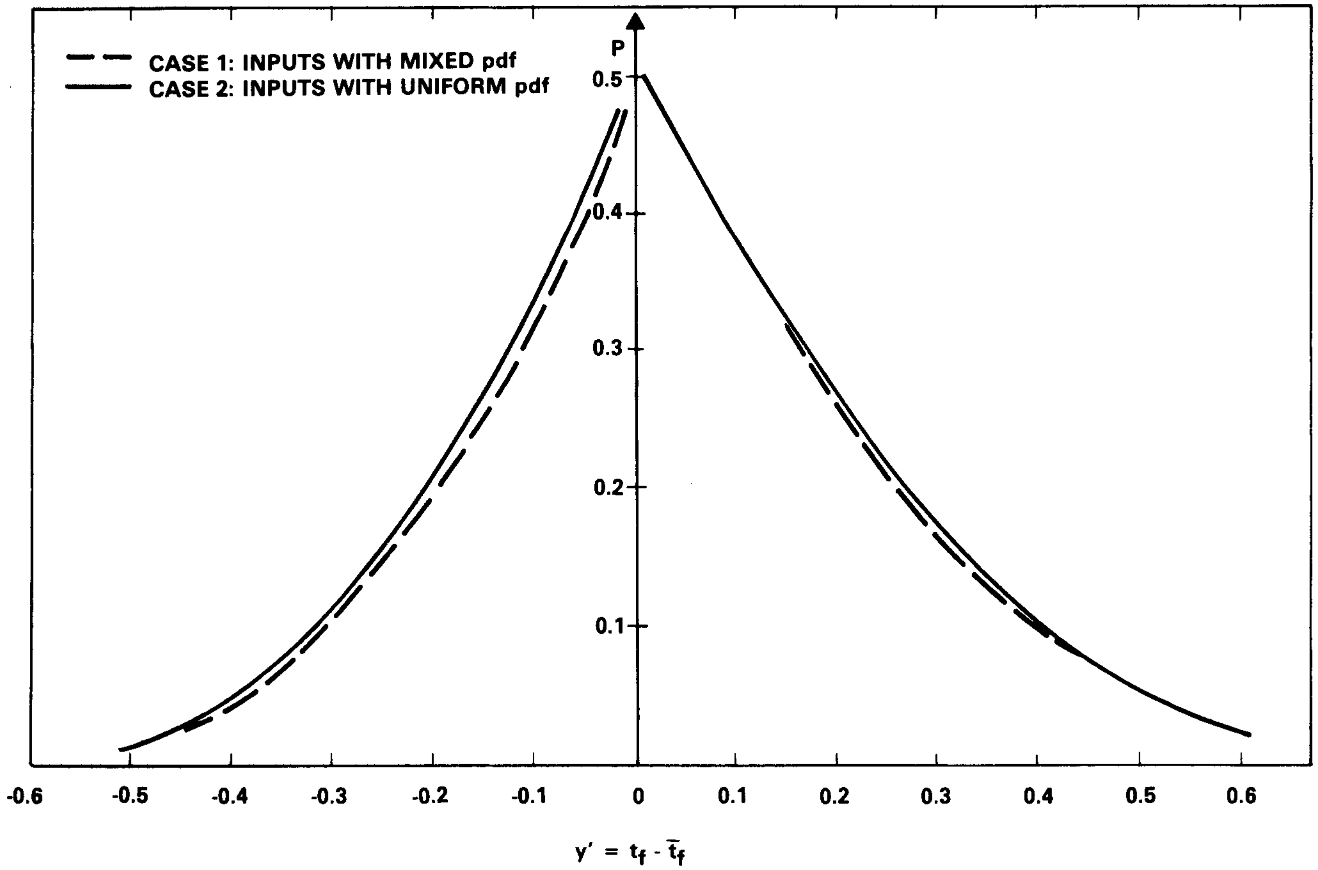

Fig. 2. Probability functions P^- and P^+ for different combinations of input variables pdf (σ_i = 5 to 10% of nominal values).

$$p(x) = \frac{1}{2h} \ , \quad |x| \leqslant h \tag{1}$$

are[5]

$$\mu_r = \frac{h^r}{(r+1)} \ , \quad r \text{ odd}$$
$$= 0 \ , \qquad r \text{ even} \ . \tag{2}$$

For $\sigma = 1$, $h = \sqrt{3}$. When calculations are performed using correct moments, the distributions are obtained as shown in Fig. 1. They do not differ significantly, and indeed the distribution for the "mixed" inputs case is slightly more peaked than that for the "uniform" inputs case, as observed by Cox et al.[1] Naturally, the largest error occurred in the "uniform" case of Fig. 3 of Ref. 2. However, the major source of the problem is the error of the moments used for the rectangular distribution, not the approximate nature of the methods of error propagation followed by moment matching. This, however, is not purported to imply that the potential errors introduced by these approximate methods are always controllable. Reference 2 did not address the error assessment of the moment matching techniques.

I believe that these techniques are economical and have useful applications, especially when exact results cannot be obtained. The difficulty in the analytic reconstruction of a distribution from its infinite sequence of moments is too well known, even when these moments are available. How accurately could continuous distributions be obtained from the Monte Carlo histograms of Figs. 1 and 2 of Ref. 1? Starting with Eq. (13) of Ref. 2, it took only ~26 s of computer processing time in a UNIVAC-1100 to get the results of Figs. 1 and 2, an advantage that sometimes becomes an overriding consideration in the uncertainty analysis of large codes.

The results obtained from the cases considered here cannot be generalized to all types of input distributions. The small sensitivity of output distributions to input distributions observed above is mainly due to certain similarities between rectangular and normal distributions: both are symmetric and unimodal about zero. In general, I suspect this sensitivity to be dependent on the shapes of the input distributions (symmetric versus asymmetric, bimodal versus unimodal), on the way these distributions are combined (i.e., physical modeling), and on the number of inputs considered in the problem. The matter requires further study.

Finally, Fig. 2 shows the probability functions $P-$ and $P+$ using the distributions of Fig. 1. The range of the calculated failure time at 95% confidence level is 2.636 s $\leqslant t_f \leqslant$ 3.696 s, instead of 2.656 s $\leqslant t_f \leqslant$ 3.646 s as reported in Ref. 2.

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