FURTHER COMMENTS ON "THE UNCERTAINTY IN ACCIDENT CONSEQUENCES CALCULATED BY LARGE CODES DUE TO UNCERTAINTIES IN INPUT"

The paper by Nguyen[1] and the subsequent correspondence[2] raise a confusing issue that might be clarified by the rigorous application of some simple concepts from probability theory. It seems to me that Nguyen's concern about the sensitivity of output probability density functions (pdf's) to the form of the input pdf's is justified, although Cox et al.[2] is correct in noting that such concern is unfounded for the particular situation presented in the paper. I would like to offer some simple examples in which the output pdf is quite sensitive to the input pdf's, and suggest some intuitive guidelines that might be used in applying probability theory to engineering problems.

Suppose x and y are random variables with uniform pdf's:

$$p_x = p_y = \begin{cases} \frac{1}{2} & -1 \leqslant x,y \leqslant 1 \\ 0 & \text{otherwise} \end{cases} .$$

Then, if $f(x,y) = x + y$, the output density function is given by

$$p_f(z) = \begin{cases} \frac{1}{4}z + \frac{1}{2} & \text{if} \quad -2 \leqslant z \leqslant 0 \\ -\frac{1}{4}z + \frac{1}{2} & \text{if} \quad J \leqslant z \leqslant 2 \\ 0 & \text{otherwise} \end{cases} .$$

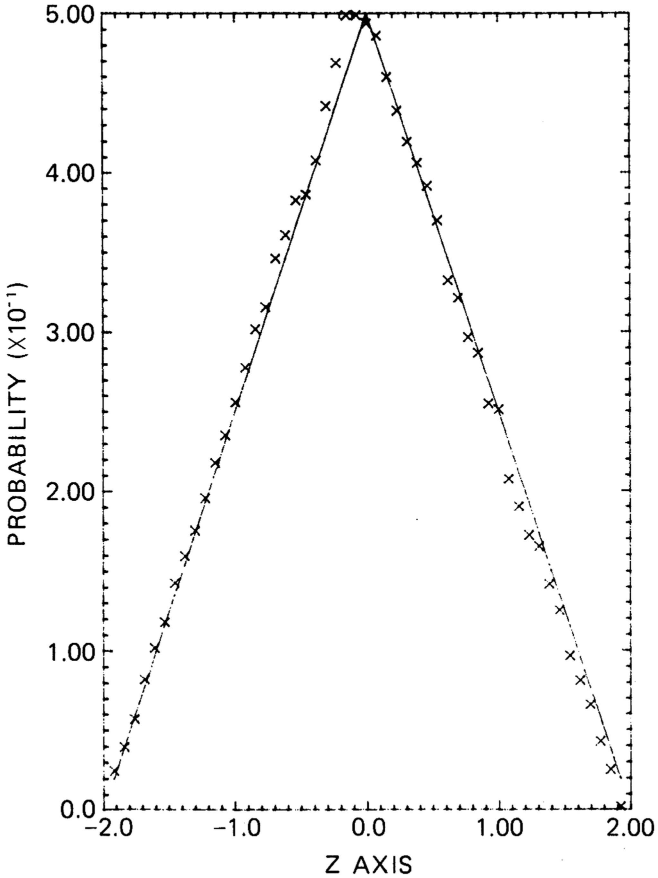

Fig. 1. Density function for sum, 100 000 trials.

Figure 1 shows the output density function given above, together with a Monte Carlo output for the same problem. Clearly, the output pdf is far from uniform or normal.

Similarly, if the above uniform random variables are operated on by the product function, $f(x,y) = xy$, then the output density function is given by

$$p_f(z) = \begin{cases} -\frac{1}{2}\ln|z| & \text{if} \quad 0 \leqslant |z| \leqslant 1 \\ 0 & \text{otherwise} \end{cases} .$$

Figure 2 shows this product density function. The slight shift to the left of the Monte Carlo outputs is due to a bookkeeping error in my code. Unfortunately, I did these runs some time ago and no longer have access to the code or similar output from the corrected code. If x and y had been normal about zero in the above examples, similar behavior near zero would be observed, and the domain of the output density functions would be the entire real line (except for the origin in the product case).

One can calculate output pdf's for progressively more complex functions and empirically observe that as the mathematical calculations become more complex and as the number of input variables increases, the sensitivity of the output pdf to the input pdf's decreases. In fact, the output pdf tends to become normal. This is also suggested by the central limit theorem. Real engineering computer calculations in general involve enough input variables that the output can be considered to be normal. This assumption should be questioned, however, if the output function has a singularity or a zero within two or three standard deviations of the mean.

In general, one assumes that the potential for pathology increases with complexity. The subject discussed here

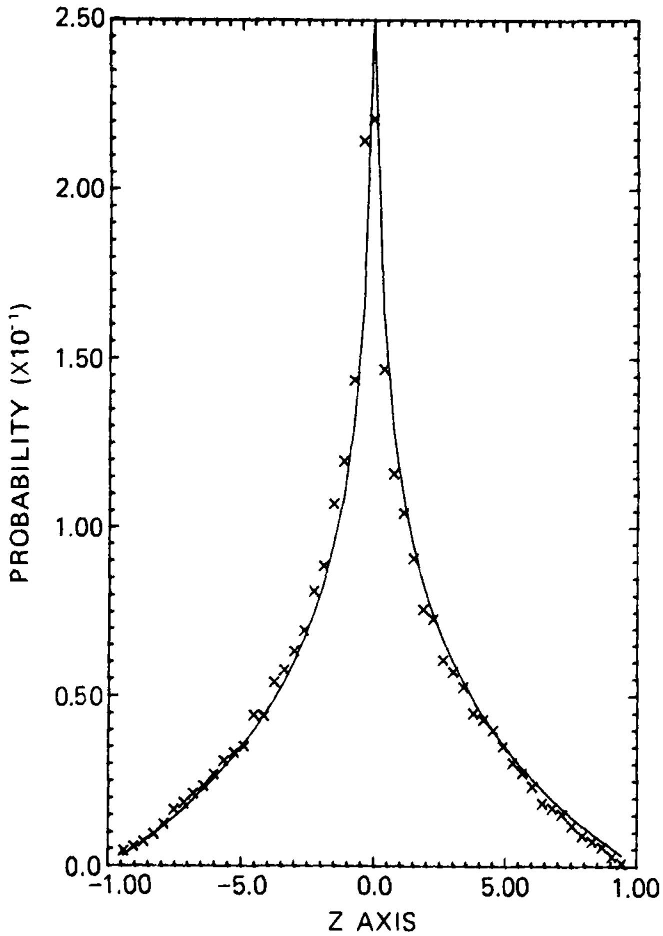

Fig. 2. Density function for product, 50 000 trials.

seems quite interesting to me because it provides a counter example to the above generalization.

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2. DAVID C. COX, PAUL BAYBUTT, and ROBERT E. KURTH, "Comments on 'The Uncertainties in Accident Consequences Calculated by Large Codes due to Uncertainties in Input,' " *Nucl. Technol.*, **52**, 439 (1981).

REPLY TO "FURTHER COMMENTS ON 'THE UNCERTAINTY IN ACCIDENT CONSEQUENCES CALCULATED BY LARGE CODES DUE TO UNCERTAINTIES IN INPUT' "

Baker's letter[1] is a welcome contribution to the understanding of the sensitivity of output probability density functions (pdf's) to the form of input pdf's.

In my earlier reply[2] to Ref. 3, it was stated that this sensitivity could depend "on the shapes of the input distributions (symmetric versus asymmetric, biomodal versus unimodal), on the way these distributions are combined (i.e., physical modeling), and on the number of inputs considered in the problem." I further stated that "the matter requires further study." Baker's letter constitutes a useful step in this direction, by addressing the last two points. His observation, that as the number of input variables increases "the output pdf tends to become normal," is of particular interest. If this number can be determined, regardless of the form of the input pdf's, for the physical situation under consideration, then a useful criterion could be established to check the results of the uncertainty analysis of large codes.

I would like to hereby reiterate the fact that the "confusing issue" relating to the particular example considered in my paper[4] was caused by an algebraic error in the moments of a pdf involved, not by the approximate nature of the moments matching technique employed in that paper.

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