REPLY TO "COMMENTS ON 'A SIMPLE RELATIONSHIP OF MAXIMUM Δk DUE TO COMPACTION OF UNMODERATED FISSILE MATERIALS' AND THE USE OF THE TROMBAY CRITICALITY FORMULA FOR THE SAME"

Comparison of k_{eff} results tabulated in the Letter[1] by Kumar and Srinivasan on my Note[2] indicates for all practical cases the equivalence of the two relationships under discussion for predicting Δk_{eff} as a function of compression. Other than a question of normalization (for absolute values of k_{eff}), my proposed 2/3 power law associating compression with k_{eff}'s (as developed in my Note, Ref. 2) can be considered as empirically confirmed and validated by the extensive numerical work of Kumar and Srinivasan et al. in Ref. 3 and their extensions on compression in Ref. 1. In fact, the question of normalization unnecessarily masks the general agreement between the Δk_{eff} results of Kumar and Srinivasan and myself, although the physical bases of our models are essentially different. The problem of normalization will subsequently be considered by an application and comparison of both methods to the ^{235}U sphere treated in Ref. 2.

It is interesting that the two models used the same basic reactivity index ($\rho R/3$ in Ref. 1, and ρR in Ref. 2) in essentially different ways deriving the same 2/3 variation law between reactivity and compression. I used[2] ρR to estimate the increase (upon compression) of neutron reaction rates, a volumetric phenomenon neglecting leakage, whereas Kumar and Srinivasan[1] and Kumar et al.[3] used ρR to estimate a nonleakage probability applied to k_∞, a surface phenomenon. In this sense, the methods are independent physically from each other. As noted in Ref. 1, the universal empirical relation (UER) reduces to my Eq. (1) of Ref. 2 when it is subjected to a Taylor series expansion of the nonleakage exponentials and when only first-order terms are retained. This would indicate that the dominant phenomenon controlling the change in k_{eff} is that due to the increased (for compression) or decreased (for dilation) internal neutron reaction rates inferring leakage to be a second-order effect. This latter point is the basis of my model in Ref. 2.

I would also like to point out a recent successful application[4] of the proposed 2/3 power law to estimating the variation of reactivity worth of bubbles in molten cores as a function of void fraction and bubble size. The results in Ref. 4 also show that the 2/3 power law is insensitive to core composition (i.e., the core k_∞). This application indicates the first-order importance of changes in fuel density (neutron reaction rates internal to the system) relative to the second-order leakage for the compression or dilation of unmoderated systems.

The results of the Table I in Ref. 1 show that, for the 16.2-kg plutonium metal case, there is general agreement between the two methods up to ~20% compression for both calculations normalized at critical. Past 20% compression, as pointed out in Ref. 2, k_{eff} is overestimated up to 10% in k for a compression factor of 2. The relationship developed by Kumar and Srinivasan agrees with KENO past this point, showing the corrective power on the fitted k_∞^* of the nonleakage term for initial high-density systems.

In Table II of Ref. 1, both the k_{eff}'s using method A or B obey the 2/3 power law. Since the method of Kumar and Srinivasan normalizes at critical, they calculate N_{crit} to be 13.1 and then proceed to generate k_{eff}'s down to $N = 1.00$. Tabulation shows the UER results to be ~25% higher than KENO, whereas the linear Eq. (10) is in better agreement. I would like to point out here that Table II gives a KENO k_{eff} of unity for N = 13.1. I performed this KENO case, compressing the PuO_2 sphere to a radius of 2.29 cm to a density of 39.8 g/cm^3. The k_{eff} was calculated to be 0.958 ± 0.006. Using my expression for $N_{crit} = (1/k_0)^{3/2}$ gave N_{crit} equal to 14.4; the KENO k_{eff} for this case was 1.026 ± 0.006. The tabulation of Table A in this Letter should be added to Table II of Ref. 1, correcting the 13.1 compression row and adding a 14.4 row.

Again, Table III of Ref. 1 for cylinders of various H/D ratios ranging from 3.0 to 0.6 shows that the k_{eff}'s calculated obey the 2/3 power law. It appears that Kumar and Srinivasan had some confusion about the cylindrical geometry used in my Note[2] concerning the cylinders of PuO_2. The four cylinders all had the same diameter (12.7 cm), with the appropriate height, considering the initial density and compression being postulated. These pertinent data are given in Table B in this Letter. Again, as before, there is excellent agreement with KENO Δk's for both models. The k_{eff}'s for the second, third, and fourth cases under A and B would therefore change from those listed in Table III when the appropriate σ's are used determined by the correct H/D.

Using the methods described in Refs. 1 and 3, I have calculated similar cases for the ^{235}U sphere I treated in Ref. 2. This case considered a 15-kg ^{235}U sphere with an original radius of 5.74 cm and a density of 18.9 g/cm^3. Table C of this Letter uses the same numbering identification as Table II of Ref. 2, but is expanded to include the new calculations similar to the tables of Ref. 1. Using the best fit for k_∞^* of 2.224, θ of 0.597, and the appropriate ratio of σ's and N's, Eq. (5a) of Ref. 1 gave the k_{eff}'s listed in the UER column of Table C; the k_{eff}'s

TABLE A

Compression, N	KENO	A, Eq. (5)	B, Eq. (10)	Marotta, Eq. (1)
13.1	0.958 ± 0.006	1.000	1.000	0.939
14.4	1.026 ± 0.006			1.000

TABLE B

Initial Density (g/cm^3)	Height (cm)		$(H/D)_{avg}$ D = 12.7 cm
	Initial	Final	
1.8	38.7	34.3	2.88
3.0	23.1	20.1	1.70
5.0	13.1	11.3	0.96
11.5	8.2	7.9	0.63

TABLE C

Case	R/R_0	N	ANISN	σ (kg/m^2)	UER	Eq. (10)	Marotta
X	0.848	1.64	---	---	---	---	1.000
Y	0.827	1.77	---	530	1.000	1.000	---
1	1.0	1.0	0.7199	362	0.745	0.683	---
2	2.0	$\frac{1}{8}$	0.1827	90.4	0.215	0.171	0.1800
3	3.0	$\frac{1}{27}$	0.0805	40.2	0.099	0.076	0.0800
4	4.0	$\frac{1}{64}$	0.0451	22.6	0.056	0.043	0.0450

determined by (1.000) $(N/N_{crit})^{2/3}$ are given in the Eq. (10) column of Table C. It is interesting that if a k_∞^* of 3.0 and θ of 0.405 were used, better agreement with ANISN is achievable for UER results, namely, 0.723, 0.2001, 0.090, and 0.049 for cases 1 through 4 in Table C. This illustrates the range of different k_{eff}'s calculated using the method of Kumar and Srinivasan from the fitted parameters in their empirical equations.

The question addressed in my Note[2] was that of calculating Δk_{eff} due to compression (or dilation) of an unmoderated system. The absolute, "quality," calculation of the initial k_0 was not considered. This is a basic neutron theory problem, as is well known, peculiar to the problem at hand to effect a quality k_0 calculation, e.g., choice of an appropriate effective neutron cross-section set with the mathematical and numerical scheme for solution of the Boltzmann equation with proper geometric approximations and modeling, adequate sampling from proper regions of space, etc. Once this quality k_0 is calculated, with the 2/3 power law established, normalization is no problem. There is no special status or importance for the critical eigenvalue (other than the desired steady state of neutron density, for which this corresponding eigenfunction has physical significance by being the persistent distribution of neutrons and hence experimentally measurable and reproducible, etc.) relative to other eigenvalues. This is so both from the eigenvalue theory of differential equations and from the theory of neutron chain reactions. All eigenvalues are meaningful and equally important physically.

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