BOOK REVIEWS

Selection of books for review is based on the editor's opinions regarding possible reader interest and on the availability of the book to the editor. Occasional selections may include books on topics somewhat peripheral to the subject matter ordinarily considered acceptable.

Finite Element Method: Basic Technique and Implementation

Authors Pin Tong and John N. Rossettos

Publisher The MIT Press (1977)

Price $19.95

Reviewer J. P. Hennart

Among the handful of books devoted to the finite element method that are published each year, this one exhibits some particular features that give it a specific flavor. First, it is one of these few books that try to bridge the gap between a purely mathematical description of the finite element method and an approach almost exclusively oriented toward civil engineers. In this reviewer's opinion, the authors have fairly well succeeded in their attempt, and the result is a book that should appeal to a broad spectrum of students, engineers, and scientists interested in the finite element method and more specifically in its practical implementation. The mathematical level remains quite (if not too) elementary throughout the book, many examples are worked out with well-known second-order ordinary or partial differential equations of mathematical physics, and the applications to solid mechanics are always preceded by a brief recall of the basic relations used. Most of the illustrative examples are treated with much detail, and, in particular, some key operations with finite element matrices, such as assembling, constraining, and solving, are spelled out again and again, introducing by the way some repetition that could well enhance the self-study features of the book, as the authors suggest in the Introduction. As in most of the finite element books, the emphasis is on the displacement model. Here, however, the hybrid-model concept, to which the authors have significantly contributed, is also introduced, and an entire chapter devoted to it; this is one of the other distinct features of this text.

After a brief sketch of the historical background, Chap. 1 consists of a physical and mathematical introduction to the basic concepts; the discretization of a given model into finite elements, the characterization of the elements, the determination of the local element matrices and their assembly into a global matrix, as well as the application of appropriate boundary conditions, are illustrated by very simple examples (which are sometimes too particular and therefore misleading in this reviewer's opinion). Nevertheless, the exposition is quite clear, and the examples are worked out in great detail. These characteristics still prevail throughout Chap. 2, where the authors go over the same basic steps for Poisson's equation, this time with piecewise linear triangular and bilinear rectangular elements.

In Chap. 3, the algebraic manipulations involved in a finite element solution are examined with some more care, in view of the large systems of equations normally found in practice. The procedures of assembling and constraining such systems are revised, and the basic direct and iterative techniques of solution are discussed, again with moderate size examples worked out explicitly. Some minimization schemes are mentioned, and a whole section is devoted to the practically important static condensation technique. The efficient implementation of assembly and solution schemes for large systems with symmetric and banded matrices is discussed in Chap. 4; the Gaussian elimination scheme is modified to take full advantage of these particular features, and some varieties of Gaussian elimination, such as the frontal method, are presented that make use of the external storage capabilities of digital computers. Still another scheme is presented that stores the matrix as a one-dimensional array, reducing even more the storage and computation requirements.

Chapter 5 presents applications to solid mechanics, restricted to the case of linear elastic materials. The basic constitutive equations and the principle of minimum potential energy are recalled, while the corresponding finite element formulations are described. Some elements, most famous among structural engineers, are discussed in detail.

In Chap. 6, higher order interpolation functions are introduced over square and right triangles. For the square

element, only the so-called serendipity family is exhibited, while the tensor-product one is hardly mentioned. More general element shapes, such as the rectangle and the general quadrilateral, the triangle, and even curved-sides elements, are derived by coordinate transformations. As a result, the triangular area coordinates are probably not introduced in the most natural way. Moreover, the presentation, which is again formally perfect, disregards some problems, such as that of a vanishing Jacobian, which are surprisingly likely to appear in practical applications if some care is not exercised.

In Chap. 7, the bending of beams and plates is examined, and it is shown that, for the corresponding fourth-order differential problems, the compatibility conditions on the interpolation functions are not easily satisfied, except in the one-dimensional case. Several possibilities of overcoming the difficulties of satisfying compatibility are sketched in this chapter, which actually serves as an introduction to Chap. 8, in which the hybrid element concept is developed as just another approach to resolve the compatibility dilemma. Chapter 9, the last one, is devoted to selected topics and recent developments, including dynamic problems of elastic solids, hybrid singular and infinite-domain super-elements, applications to heat transfer and fluid flow, etc. Four appendices on matrix algebra, rectangular and triangular elements, and variational methods close the book.

In summary, this book offers, through its first six chapters, an excellent scholarly introduction to the practical implementation of the finite element method, while more advanced topics are discussed in the last three chapters. A reader with no previous knowledge of the method would quite easily pick up the basic philosophy and the most relevant technical aspects, especially if he goes through the exercises provided with most of the chapters. Of course, the authors' approach, especially in its purposely elementary mathematical level, is not without shortcomings. In the first chapter, for instance, the completeness requirement on the basis functions is introduced in a somewhat awkward way that does not even refer to the constant strain condition, well known in the structural-mechanics-oriented literature. A correct and thorough definition of a complete polynomial in more than one variable is never given, and several subsequent developments that make use of this concept are therefore unnecessarily obscured.

A final remark this reviewer would like to make is that the references are grouped by chapter at the end of the book. As a rule, more references are offered than are referred to in the text. This would be all right, except that some of them are still to company reports, while the corresponding material should be well established in the open literature. More unfortunately, several references made in the text cannot be found among the list at the end.

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Actinides in the Environment

Author	Arnold M. Friedman
Publisher	American Chemical Society
Pages	107
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Reviewer	Reiner Papp

In addition to reactor accidents, the problems associated with isolation of nuclear wastes have become major concerns within the nuclear community and among the public. It seems very likely that analyses dealing with waste isolation could obtain, within a short time, a public awareness comparable to that of *The Reactor Safety Study*, WASH-1400. Safety assessments of waste isolation facilities focus on nuclide migration in the environment, whereby the confining ability of geologic formations for long-lived nuclides such as actinides plays an important role. It is therefore very laudable that this book provides insight into the various effects that govern mechanisms of migration of actinides in geologic formations. It is also made clear that ultimate environmental behavior of actinides is determined by their environmental chemistry.

Most contributions to the book provide evidence that the "distribution coefficient," K^d, which relates concentrations of radionuclides in solid and liquid phases (water), is the most important factor. It describes the sorption properties based on ion-exchange of a nuclide relative to each type of soil. Without sorption, even an excellent confining formation can only delay the "breakthrough" of chemical substances (e.g., chlorides, iodine). On the other hand, for reactive materials such as some actinides, this coefficient may be quite large. One contribution to the book discusses the nuclide migration in geologic barriers surrounding the 1.8-billion-year-old natural fission reactor in Oklo (Gabun). Studies showed little evidence that uranium, neptunium, and plutonium have moved any detectable distance in the Oklo formation. On the other hand, migration of plutonium over considerable distances in the ground has been reported recently at Maxey Flats (Kentucky). Under certain circumstances, some chemical complexes of plutonium seem to show little ion-exchange (and sorption).

Although the book provides some specialized contributions, it seems to be readable for everyone who is interested in a deeper understanding of confinement properties of waste isolation facilities. The book certainly is a very valuable reference work for those who are involved in risk analyses of the nuclear fuel cycle.

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