LETTERS TO THE EDITOR

BREEDING RATIO FOR FAST REACTORS

Dear Sir:

At the last American Nuclear Society Winter Meeting "An Improved Definition of the Breeding Ratio for Fast Reactors" was suggested by K. O. Ott.[1] The general definition of the breeding ratio is

$$\mathrm{BR} = \frac{\sum_i \gamma_i C_{i-1}}{\sum_i \gamma_i A_i} \quad , \tag{1}$$

where i is a suffix that can take the values 5, 6, 8, 9, 0, 1, and 2 representing ^{235}U, ^{236}U, ^{238}U, ^{239}Pu, ^{240}Pu, ^{241}Pu, and ^{242}Pu, respectively; C_i, A_i, and F_i [in Eq. (4)] denote, respectively, the capture, absorption, and fission rate for isotope i integrated over the whole reactor; and the γ_i are weighting factors. The improved weighting factors suggested in Ref. 1 are

$$\gamma_i = \frac{\eta_i}{\eta_9} \quad \text{with} \quad \eta_i = \frac{\nu_i \sigma_{fi}}{\sigma_{ai}} \quad . \tag{2}$$

From these definitions it is possible to find the reactor breeding gain

$$G_\eta = \mathrm{BR}_\eta - 1 = \frac{\sum_i \gamma_i (C_{i-1} - A_i)}{\sum_i \gamma_i A_i} \quad . \tag{3}$$

Let us write the denominator of Eq. (3) as follows:

$$\begin{aligned} \sum_i \gamma_i A_i &= \sum_i \frac{1}{\eta_9} \frac{\nu_i \sigma_{fi}}{\sigma_{ai}} A_i = \sum_i \frac{\nu_i}{\eta_9} F_i \\ &= \left\langle \frac{\nu}{\eta_9} \right\rangle \sum_i F_i \quad , \end{aligned} \tag{4}$$

where

$$\left\langle \frac{\nu}{\eta_9} \right\rangle = \frac{\sum_i \frac{\nu_i}{\eta_9} F_i}{\sum_i F_i} \quad . \tag{5}$$

From Eqs. (3) and (4), one gets

$$\begin{cases} G_\eta = \dfrac{\sum_i g_i (C_{i-1} - A_i)}{\sum_i F_i} \\[2ex] g_i = \dfrac{\gamma_i}{\left\langle \dfrac{\nu}{\eta_9} \right\rangle} \quad . \end{cases} \tag{6}$$

Now let us turn to the so-called British definition[2,3] of the breeding gain

$$\begin{cases} G_{\mathrm{UK}} = \dfrac{\sum_i w_i (C_{i-1} - A_i)}{\sum_i F_i} \\[2ex] w_i = \dfrac{(\nu\sigma_f - \sigma_a)_i - (\nu\sigma_f - \sigma_a)_8}{(\nu\sigma_f - \sigma_a)_9 - (\nu\sigma_f - \sigma_a)_8} \quad . \end{cases} \tag{7}$$

For a typical fast reactor spectrum, one has

Isotope i	^{239}Pu	^{240}Pu	^{241}Pu	^{242}Pu
w_i	1.0	0.08	1.50	0.10
g_i	0.82	0.38	0.89	0.42
$\gamma_i \approx g_i/g_9$	1.0	0.46	1.09	0.52

Clearly the breeding ratio will be less sensitive to the plutonium isotopic composition with the weighting factors g_i than with w_i.

As stated by Ott, if the breeding gain was insensitive with respect to changes in the plutonium composition it is true that it would allow a good estimate of the equilibrium breeding gain and doubling time using only static reactor calculations. But unfortunately it is not insensitive, and there is no apparent physical ground to define the weighting factors as in Eq. (2).

As a matter of fact, the w_i are the relative reactivity worths of the different isotopes, and extensive fast reactor physics calculations have shown that the critical mass for a given reactor is practically constant when expressed in equivalent ^{239}Pu.

The British definition of the breeding gain [Eq. (7)] is precisely the one that should be used to calculate the reactor doubling time. Indeed, the standard definition of the doubling time of a given reactor is the time needed to build up an amount of fuel material sufficient to make another similar reactor critical.

Such a definition of the doubling time cannot be readily derived from the definition of the breeding ratio given by Eq. (6).

For instance, if a fast breeder reactor is supposed to utilize plutonium with a very high ^{239}Pu content, it can be seen from Fig. 1 of Ref. 1 that

$$\mathrm{BG}_\eta \approx 0.225 \quad ; \quad \mathrm{BG}_{\mathrm{UK}} \approx 0.10 \quad .$$

The doubling time derived from BG_η will then be, in this case, less than half that found with BG_{UK}. This shows the importance of a consistent definition of the breeding gain.

Of course, using Eq. (7), one must be careful and use the best average equilibrium fuel composition.

If the composition of reloaded fuel varies sharply with time, one should use

$$\langle BG_{UK} \rangle = \frac{1}{t} \int_0^t BG_{UK}(t)\, dt \qquad (8)$$

to compute the doubling time.

Concerning the definition of the doubling time, one can state more precisely that to start up a reactor one needs an initial fuel mass which is more than critical, to compensate for the loss of reactivity during burnup. Since this loss of reactivity depends strongly on the fuel isotopic composition it is possible to calculate adequately the corresponding weighting factors w_i as was shown in Ref. 4. But there is then no simple definition of the weighting functions w_i as in Eq. (7), since they include not only the physical properties of an isotope but also those of all its daughter isotopes (produced during irradiation), and some economic factors too.

One might also define an "economic doubling time" as the time needed to build up an amount of heavy isotopes the sale value of which is equal to the purchase cost of a more than critical mass sufficient to start up a similar reactor.

The weighting coefficients would then be

$$(w_i)_{\text{econ}} = \frac{P_i}{P_9} \quad , \qquad (9)$$

where P_i is the price for isotope i. Such prices P_i depend on the evolution of the world market and not on the kind of reactor we are considering. However, the practical interest of such a definition is questionable as, presently, most of such prices are not fixed.

P. Goldschmidt

Belgonucleaire
35 Rue des Colonies
Bruxelles 1000, Belgium

March 18, 1970

REFERENCES

1. K. O. OTT, *Trans. Am. Nucl. Soc.*, **12**, 719 (1969).

2. A. R. BAKER and R. W. ROSS, "Comparison of the Value of Plutonium and Uranium Isotopes in Fast Reactors," ANL-6792, Argonne National Laboratory (October 1963).

3. A. R. BAKER, Letter to the Editor, *Nuclear News* (April 1967).

4. L. HERMANS and M. EGLÈME, "Valeur d'Equivalence des Divers Isotopes de l'Uranium et du Plutonium dans les Réacteurs à Neutrons Rapides," *IAEA Symp. on Economics of Nuclear Fuel, Gottwaldov,* International Atomic Energy Agency (May 1968).