so that for a physical wave function, as $a \rightarrow 0$, the solution corresponding to s_- does not contribute. However, because of the finite size of the nucleus, the wave function consists of a large component corresponding to s_+ and a small component corresponding to s_- with a binding energy that is thus very close to the original binding energy $E_+ - mc^2$. For instance, for the case of a proton, the proton radius $a \approx 1$ fm, $Z = 1$, and $E \approx E_+ = mc^2 - 13.6$ eV; therefore, $\alpha \approx 3.78 \times 10^{-5}$ fm^{-1}, and hence, $C/B \approx -0.2 \times 10^{-8}$.

Furthermore, we note that their electron orbits of radius $\sim 5 \times 10^{-13}$ cm are 50 times smaller than muonic orbits of 250×10^{-13} cm. If such orbits existed, upon collision, they would produce fusion at a much higher rate than muon-catalyzed fusion. So, their proposed solution does not get around the nuclear ash problem.

In summary, we have shown that the deep Dirac orbits do not exist. They are an artifact due to the incorrect use of the irregular solution of the relativistic Dirac equation and the total neglect of the regular solution. Furthermore, even if such orbits existed, this does not keep the problem in the domain of exotic chemistry with the avoidance of nuclear effects and the nuclear ash problem. Such tight orbits would be expected to produce considerably high fusion rates.

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RESPONSE TO "COMMENTS ON 'ELECTRON TRANSITIONS ON DEEP DIRAC LEVELS I'"

The central point of our response to Ref. 1 is as follows. Let us assume that $\psi_n(x)$ is a solution of the time-independent portion of the Schroedinger equation for energy E_n. The authors of Ref. 1 indicate that if $\psi_n(x)$ and $\psi_m(x)$ are solutions, then $A\psi_n(x) + B\psi_m(x)$ is also the solution, thus completely disregarding the fact that n and m are two different quantum states with different energies E_n and E_m; $\psi_n(x)$ and $\psi_m(x)$ are the solutions of two different differential equations. The complete solution of the time-dependent Schroedinger equation for a particle in the quantum state $\psi_n(x)$ of energy E_n is

$$\psi_n(x,t) = \exp[-i(E_n/\hbar)t]\psi_n(x) . \tag{1}$$

The probability distribution $\psi^* \cdot \psi$ is independent of time; thus, the particle stays in the energy state E_n indefinitely. Therefore, it is not a portion of time in the energy state E_m, as the authors indicate when they write the wave function $[A\psi_n(x) + B\psi_m(x)]$.

Specifically, our response to their criticism is the following:

1. The authors[1] combine in a linear combination two solutions corresponding to s^+ and s^- parameters. They correspond to two different quantum states, which, in turn, correspond to two different differential equations and two different energy levels. To use the linear combination $\psi = B\rho^{s^++1} + C\rho^{s^-+1}$ as a general wave function would be correct only if both terms $B\rho^{s^++1}$ and $C\rho^{s^-+1}$ would be the solution of the same differential equation. However, this is not the case. The equation, which has to be solved, is Eq. (9) in Ref. 2:

$$\rho^2 L'' + \rho[2(s+1) - \rho]L'$$
$$+ [\rho(\lambda - s - 1) + s(s+1) - l(l+1) + \gamma^2]L = 0 . \tag{2}$$

This equation for $\rho = 0$ is zero only if it is satisfied:

$$s(s+1) + \gamma^2 - l(l+1) = 0 , \tag{3}$$

which gives the solution [see Schiff[3] Eq. (51.19) or Eq. (10) in Ref. 2]:

$$s = -\tfrac{1}{2} \pm [(l + \tfrac{1}{2})^2 - \gamma^2]^{1/2} . \tag{4}$$

The aforementioned s^+ and s^- correspond to two different signs in Eq. (4). Because s^+ and s^- are clearly different, Eq. (2) with inserted $s = s^+$ is different from Eq. (2) with inserted $s = s^-$; the resulting energy levels E_{s^+} or E_{s^-} are different for s^+ or s^- [see Eqs. (12) and (5) of Ref. 2]. Thus, one cannot use $\psi = B\rho^{s^++1} + C\rho^{s^-+1}$ as the most general wave function [$B\rho^{s^++1}$ contains E_{s^+} and $C\rho^{s^-+1}$ contains E_{s^-} inside $\rho = \alpha(E)r$].

2. Equations (9) of Ref. 1 define the potential inside and outside the nucleus:

$$e\phi(r) = \begin{cases} -Ze^2/a & \text{for } r \leq a \\ -Ze^2/r & \text{for } r > a . \end{cases} \tag{5}$$

The potential outside the nucleus is correct, but the potential inside the nucleus is only an approximation. However, two errors appear in Eq. (10) of Ref. 1:

$$\chi(r) = rR(r) = \begin{cases} AKr & \text{for } r \leq a \\ B\rho^{s^++1} + C\rho^{s^-+1} & \text{for } r > a , \end{cases} \tag{6}$$

where $\hbar^2 c^2 K^2 = (E + Ze^2/a)^2 - m^2c^4$ and $l = 0$. First, the correct solution of the differential equation for $r \leq a$ is $\chi(r) = A\sin(Kr)$ and not AKr, as used in Eqs. (10) and (12) of Ref. 1 [this can be derived from Eq. (51.14) of Schiff[3] by using $l = 0$ and the potential of Eq. (5)]. Second, as we explained earlier, for $r > a$, the solutions are either $\chi(r) = C_1 rR_1(\rho)$ for $s = s^+$ or $\chi(r) = C_2 rR_2(\rho)$ for $s = s^-$ [$R(\rho)$ is defined by Eqs. (7) and (8) of Ref. 2] and not a linear combination of these two solutions as shown in Eq. (6).

3. The authors of Ref. 1 have also not realized that the variable $\rho = \alpha(E)r$ contains energy levels E_{s^+} or E_{s^-} inside parameter $\alpha(E)$ [see Eq. (5) of Ref. 2], and ρ in Eq. (6) is not the same for s^+ or s^- states.

4. If one matches the logarithmic derivatives from Eq. (6) with the aforementioned corrections, one obtains relations that have completely eliminated normalization constants A, C_1, or C_2:

$$\frac{1}{\chi}\frac{d\chi}{dr}\bigg|_{r=a^-} = K\cot g(Ka) = \frac{1}{\chi}\frac{d\chi}{dr}\bigg|_{r=a^+}$$

$$= \frac{1}{a} + \frac{C_1}{C_1}\left(\frac{\frac{dR_1}{dr}}{R_1}\right)_{r=a} \quad \text{for } s = s^+$$

and

$$\frac{1}{\chi}\frac{d\chi}{dr}\bigg|_{r=a^-} = K\cot g(Ka) = \frac{1}{\chi}\frac{d\chi}{dr}\bigg|_{r=a^+}$$

$$= \frac{1}{a} + \frac{C_2}{C_2}\left(\frac{\frac{dR_2}{dr}}{R_2}\right)_{r=a} \quad \text{for } s = s^- . \quad (7)$$

As a result, one cannot calculate any relation between C_1 and C_2 [or B and C of Eq. (6)] as presented in formulas (11) and (12) of Ref. 1.

There is a way to add different wave functions describing different energy states. This is usually done to describe photon transitions between different energy states. In this case, the most general solution of the time-dependent Schroedinger equation can be written as

$$\psi(x,t) = \sum_n C_n \exp[-(E_n/\hbar)t]\psi_n(x) . \quad (8)$$

Perhaps, the authors are confused by the form of Eq. (8) when forming their argument. They have not realized that Eq. (8) includes all the possible quantum states.

In summary, Ref. 1 is completely in error because based on formulas (10), (11), and (12) in Ref. 1, one cannot determine any ratio between the constants C and B. Formula (10) of Ref. 1 [or Eq. (6) of this letter] is simply wrong.

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