

experimentally determined surface tension values is open to question because of the difficulties encountered in measuring surface tensions of liquid metals, and (b) for many metals, estimated surface tensions reported in the literature were used because of the absence of experimental values. Nevertheless, the energy of vaporization per unit volume has emerged as bearing an important relationship to the magnitude of surface tensions of liquid metals.

Data for $S(\varrho)$ and γ were obtained from the literature, and Table I gives references to the sources of the surface tension data used. Taylor's estimated values (1) were used in most cases where experimental data were not available. In some cases surface tensions at the melting point were obtained by extrapolation on surface tension-temperature plots. Figure 1 is a plot of $\log \gamma$ against $\log S$. This plot could not be improved by using, as Hildebrand and Scott did, $\log(\gamma/V^{1/3})$ for the ordinate. The data show that with but few exceptions the plotted points, which include estimated as well as experimentally determined surface tensions, fall on a straight line with relatively little scatter. It is interesting to note that the elements magnesium, zinc, cadmium, and mercury, whose plotted points show the largest deviations are metals that have small liquid ranges.

The results of this investigation indicate that surface tension and energy of vaporization per unit volume are dependent on the same basic property.

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Generalized Resonance Integral Representation*

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Chernick and Vernon (1) and Dresner (2) have pointed out that for thorium-232 and uranium-238 selective use of Narrow Resonance and the Narrow Resonance Infinite Absorber approximations can give good representations of neutron spectrum effects in the resonances.

For homogeneous mixtures of resonance absorber and scatterer the formulas for these two approximations are as given in Eqs. (1) and (2), for the N.R. and N.R.I.A. approximations, respectively.

$$R = \int \frac{dE}{E} \sigma^A(E) / \{1 + [\sigma^A(E) + \sigma^S(E)]/\sigma^P\} \quad (1)$$

$$R = \int \frac{dE}{E} \sigma^A(E) / \{1 + [\sigma^A(E) + \sigma^S(E)]/[\sigma^P - \sigma_{\text{abs}}^P]\} \quad (2)$$

$\sigma^A(E)$ and $\sigma^S(E)$ are the microscopic resonance absorption and scattering cross sections of the absorber nuclei; σ^P is the potential scattering per absorber atom of the medium; σ_{abs}^P is the potential scattering per atom of the absorber.

For lumped absorbers Chernick and Vernon simplified the spatial distribution problem through use of lump collision probabilities calculated from Wigner's rational approximation. Algebraic manipulation of Chernick and Vernon's results gives, for lumped absorbers, Eq. (3) for the N.R. approximation and Eq. (4) for the N.R.I.A. approximation.

$$R = \int \frac{dE}{E} \sigma^A(E) / \{1 + [\sigma^A(E) + \sigma^S(E)]/[\sigma^P + S/4NV]\} \quad (3)$$

$$R = \int \frac{dE}{E} \sigma^A(E) / \left\{ \frac{1 + [\sigma^A(E) + \sigma^S(E)]}{[\sigma^P + S/4NV - \sigma_{\text{abs}}^P]} \right\} \quad (4)$$

S/V is the surface-to-volume ratio of the lump; N is the number density of absorber atoms within the lump; σ^P in this case is the potential scattering within the lump per absorber atom within the lump.

Aside from dependence on quantities determined purely by the species of the absorber (i.e., whether it is U^{238} or Th^{232} , etc.) the resonance integral is a function solely of the quantity

$$(\sigma^P + S/4NV)$$

For lattices, Rothenstein (3) has shown that the shadowing effect of lumps on each other can be represented to first order by a reduction of the surface S to an effective surface S^{eff} through the Dancoff correction (4)

$$S^{\text{eff}} = S(1 - C) \quad (5)$$

Consequently it is proposed that for each species of resonance absorber a single relationship of the form

$$R = f(\sigma^P + S^{\text{eff}}/4NV) \quad (6)$$

should accommodate all cases—homogeneous mixtures, single lumps and lattices of metal, oxide, carbide, etc. This is tested in the accompanying plots of R vs $(\sigma^P + S^{\text{eff}}/4NV)$.

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$4NV)^{1/2}$ for U^{238} and Th^{232} (Figs. 1 and 2). The degree to which all measurements fall on a single curve is the measure of the success of the proposed rule. (The straightness of the curve is irrelevant for the present discussion.)

It seems likely that the lack of agreement between measurements in lattices and in single lumps is due only partly to deficiency of the present reduction scheme. When

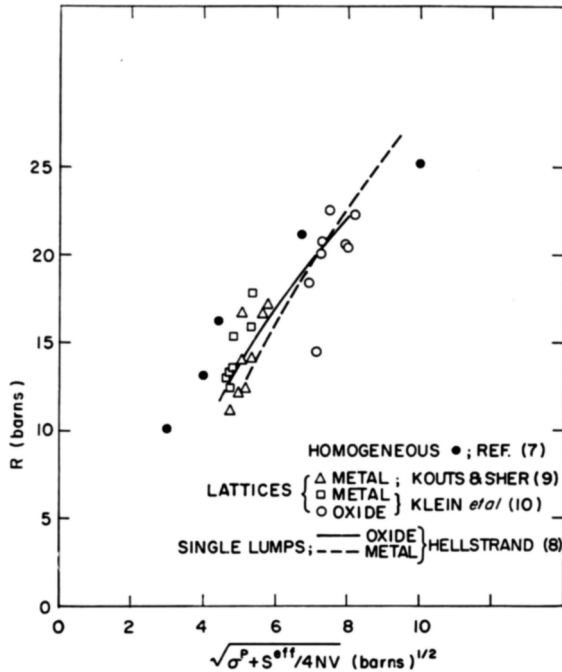


FIG. 1. U^{238} resonance integral

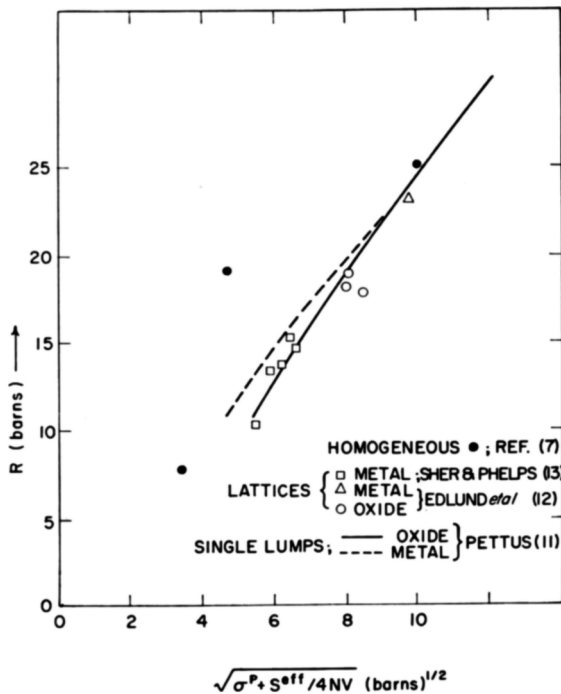


FIG. 2. Thorium resonance integral

the experimental results are analyzed for U^{238} resonance integral, some of them show peculiar behavior: (i) change of U^{235} enrichment gives an apparent change in U^{238} resonance integral; (ii) loosening a lattice with no change in rod size or density gives an apparent decrease in U^{238} resonance integral. These effects are clearly false so it appears that the quoted experimental uncertainties are not indicative of the true errors in the results.

Likewise the measured homogeneous resonance integrals show a great deal of scatter so that their errors are also likely to be larger than the nominal experimental uncertainties. Lack of agreement between measured homogeneous resonance integrals and values calculated from resonance parameters reflects additional doubt on these measurements.

Disagreement between measured resonance integrals of single lumps of metal and of oxide is more serious. This has already been pointed out by Dresner (2).

Similarity of the present results to a scaling rule introduced by Dresner (2) and to Bakshi's equivalence principle (5) should be observed. Because the approximations leading to Eqs. (3) and (4) are used by Vernon (6) in his calculations of U^{238} resonance integrals for metal, oxide, and carbide, his calculated values would coincide identically on plots of the present type.

The fractional change in resonance integral with temperature— $(1/R)(dR/dT)$ —as calculated from resonance parameters by use of Eqs. (3) and (4) is only slightly sensitive to the value of $\sigma^p + S^{eff}/4NV$ in the range of interest for most reactor lattices and single rod sizes. This indicates, for example, that Doppler effects measured for an isolated rod can be carried over to lattices.

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