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LETTER TO THE EDITORS

Two Group Calculation of Prompt Neutron Lifetime

A simple formula for the calculation of l^*, the prompt neutron lifetime, is derived from two group reactor theory. It is pointed out that the group neutron velocity (v) which appears in the equations should be calculated by inverting the average of $1/v$.

A simple formula is presented below which has proven useful in calculating the lifetime of a prompt neutron. This quantity, often called l^*, is considered to be the sum of the time required for a neutron to slow down from fission energies to thermal and the time spent in diffusion before capture as a thermal neutron. The method is based on two group theory and should thus be restricted to thermal reactors in which this theory will be valid.

The starting point is a perturbation theory formula derived by the method of Pigford *et al.* (*1*)

$$l^* = \frac{\int_V \left(\frac{\phi_1^* \phi_1}{v_1} + \frac{\phi_2^* \phi_2}{v_2} \right) dV}{\int_V \phi_1^* (\nu\Sigma_1^f \phi_1 + \nu\Sigma_2^f \phi_2)\, dV}. \tag{1}$$

In this formula ϕ_1 and ϕ_2 are the fast and thermal group fluxes, respectively; the ϕ^* are adjoints; the $\nu\Sigma^f$ are neutrons per fission multiplied by macroscopic fission cross section; and the v are the group average velocities.[1] l^* is the prompt neutron lifetime in seconds.

If we assume a reactor poisoned by a control cross section (added to Σ_2^a) to $k_{\text{eff}} = 1.00$, the two group fluxes and adjoints are independent of time and obey the following equations:

$$(\nabla \cdot D_1 \nabla - \Sigma_1^a - \Sigma_1^r + \nu\Sigma_1^f)\phi_1 + \nu\Sigma_2^f \phi_2 = 0 \tag{2a}$$

$$(\nabla \cdot D_2 \nabla - \Sigma_2^a)\phi_2 + \Sigma_1^r \phi_1 = \tag{2b}$$

$$(\Delta \cdot D_1 \nabla - \Sigma_1^a - \Sigma_1^r + \nu\Sigma_1^f)\phi_1^* + \Sigma_1^r \phi_2^* = 0 \tag{3a}$$

$$(\nabla \cdot D_2 \nabla - \Sigma_2^a)\phi_2^* + \nu\Sigma_2^f \phi_1^* = 0. \tag{3b}$$

In these equations Σ^a is the group macroscopic absorption cross section; Σ_1^r is the macroscopic cross section for removal from group 1 to group 2 (slowing down); while the D are the usual group diffusion coefficients.[2]

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[1] Actually, one should average $1/v$ over the group spectrum, as $(1/v)(\partial\phi/\partial t)$ appears on the right of the group equations.

[2] This notation is that used in many multigroup codes. A transformation to more usual two group notation is the following, which preserves the value of k_{eff} in a bare core and satisfies the definition of ϵ:

$$\Sigma_1^r = \left(\frac{\nu - 1}{\nu\epsilon - 1} \right) p\epsilon \frac{D_1}{\tau}$$

If we assume the equivalent bare, uniform reactor instead of our actual reflected core, then all fluxes and adjoints will have the same spatial form and will differ only by constants. This spatial form will be the lowest eigenfunction of the chosen geometry, and we will have:

$$\nabla^2\phi = -B^2\phi; \qquad \nabla^2\phi^* = -B^2\phi^* \tag{4}$$

$$\left.\begin{matrix}\text{critical} \\ \text{condition}\end{matrix}\right\} \quad \{\nu\Sigma_2{}^f\Sigma_1{}^r = (D_1B^2 + \Sigma_1{}^a + \Sigma_1{}^r - \nu\Sigma_1{}^f)(D_2B^2 + \Sigma_2{}^a) \tag{5}$$

$$\phi_1{}^* = \phi_2{}^* \frac{\Sigma_2{}^a(1 + L^2B^2)}{\nu\Sigma_2{}^f} \tag{6}$$

$$\phi_1 = \phi_2 \frac{\Sigma_2{}^a(1 + L^2B^2)}{\Sigma_1{}^r}. \tag{7}$$

To obtain Eqs. (6) and (7) we have used the relationship $L^2 = D_2\,\Sigma_2{}^a$.

Substitution of 6 and 7 into Eq. (1) allows a cancellation of the integral of $\phi_2{}^*\phi_2\,dV$, leaving an expression in terms of core constants alone.

$$l^* = \frac{\dfrac{1}{v_1}\dfrac{\Sigma_2{}^a(1 + L^2B^2)}{\Sigma_1{}^r\nu\Sigma_2{}^f} + \dfrac{1}{v_2}\dfrac{1}{\Sigma_2{}^a(1 + L^2B^2)}}{1 + \dfrac{\nu\Sigma_1{}^f\Sigma_2{}^a(1 + L^2B^2)}{\Sigma_1{}^r\nu\Sigma_2{}^f}}. \tag{8}$$

Use of the transformation of footnote 2 allows Eq. (8) to be recast in terms of more familiar constants:[3]

$$l^* = \frac{1}{v_1}\left(\frac{1}{1 + \tau B^2}\right)\frac{1}{D_1/\tau} + \frac{1}{v_2}\left(\frac{\nu - 1}{\nu\epsilon - 1}\right)\left(\frac{1}{1 + L^2B^2}\right)\frac{1}{\Sigma_2{}^a}. \tag{9}$$

In applying Eq. (9) to reactor calculations, several points should be borne in mind:

(a) $\Sigma_2{}^a$ should include the control cross section necessary to achieve $k_{\text{eff}} = 1$.

(b) B^2 should be the geometric buckling of the equivalent bare reactor, including reflector savings and extrapolation length.

(c) v_1 should be obtained by averaging $1/v$ over the spectrum of the fast group. One method of obtaining this spectrum is to use the MUFT code on the IBM 704 (*2*).

(d) v_2 should be obtained by averaging $1/v$ over the spectrum of the thermal group. If one assumes a Maxwellian spectrum, v_2 is 0.887 v_{kT}, *not* v_{kT} or 1.128 v_{kT}. A more sophisticated spectrum can be obtained from the SOFOCATE code for the 704 (*3*) which includes spectrum hardening effects as well as the slowing-down tail. One should also use a spectrum corrected $\Sigma_2{}^a$ if v_2 contains this correction.

$$\Sigma_1{}^a = (D_1/\tau) - \Sigma_1{}^r$$

$$\nu\Sigma_1{}^f = \left[\frac{\nu(\epsilon - 1)}{\nu\epsilon - 1}\right]\left(\frac{p\epsilon}{1 + L^2B^2}\right)\left(\frac{D_1}{\tau}\right)\left(\frac{\nu\Sigma_2{}^f}{\Sigma_2{}^a}\right)$$

[3] Criticality has also been assumed, so that

$$k_{\text{eff}} = 1 = \frac{p\epsilon}{(1 + L^2B^2)(1 + \tau B^2)}\frac{\nu\Sigma_2{}^f}{\Sigma_2{}^a}.$$

Ordinarily the whole question of spectrum effects in the slow group is not too important in l^* as $\Sigma_2{}^a$ is close to a $1/v$ dependence and $v_2\Sigma_2{}^a$ will be relatively insensitive to neutron temperature.

REFERENCES

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