NUCLEAR SCIENCE AND ENGINEERING: 1, 438–440 (1956)

LETTER TO THE EDITORS

Self-Shielding of Lumped-Poison Mixtures

It is frequently desirable in constructing a reactor to introduce poison material as an inhomogeneous mixture or suspension in some other substance rather than in the form of the pure absorbing material. In the present note, we show how to extend self-shielding factors previously calculated (*1*) for homogeneous mixtures of various geometrical shapes to the case of inhomogeneous mixtures of poison with other material, under the assumption that the poison is present in low volume concentration. For this assumption the result is quite simple: the self-shielding factor of the mixture is calculated in the usual way, except that the macroscopic absorption cross section of the poison material is replaced with the self-shielded cross section of the poison lumps in the mixture.

To prove this, consider the lumps of poison all to have equal sizes and shapes (if there is a distribution of sizes and shapes, the result will merely have to be averaged over the same). Then, the probability that a neutron incident on the edge of the mixture will be transmitted is given by

$$P_T = \langle e^{-\sigma l} \rangle_{\text{Av}} = \int_0^\infty P(l) e^{-\sigma l}\, dl, \tag{1}$$

where σ is the macroscopic cross section of the poison, l is the path length *in the poison lumps*, and the average is taken over all distributions of path lengths $P(l)$ for an isotropically incident neutron distribution.

Let S be the total path length through the mixture for a neutron incident upon the surface at a particular angle. If this is divided up into a number of segments ΔS_i, we have

$$\langle e^{-\sigma l} \rangle_{\text{Av}} = \left\langle \exp\left(-\sigma \sum_{\Delta S_i} l_i\right)\right\rangle_{\text{Av}} = \left\langle \prod_i e^{-\sigma l_i} \right\rangle_{\text{Av}} \tag{2}$$

where l_i is the path length through absorber in the segment ΔS_i. Now if it is possible to choose the ΔS_i small enough so that the probability that two lumps of absorber are in one segment is small, and at the same time to choose the ΔS_i large enough so that the probability that one lump is partially in two segments is also small, then the terms of the product in Eq. (2) are independent. Then we can write

$$\langle e^{-\sigma l} \rangle_{\text{Av}} = \prod_i \langle e^{-\sigma l_i} \rangle_{\text{Av}}. \tag{3}$$

The restrictions stated above can be written for n poison spheres of radius a per unit volume of mixture

$$n \Delta S_i \pi a^2 \ll 1, \tag{4a}$$

and

$$\Delta S_i \gg a. \tag{4b}$$

These conditions can generally be satisfied in most practical reactor applications.

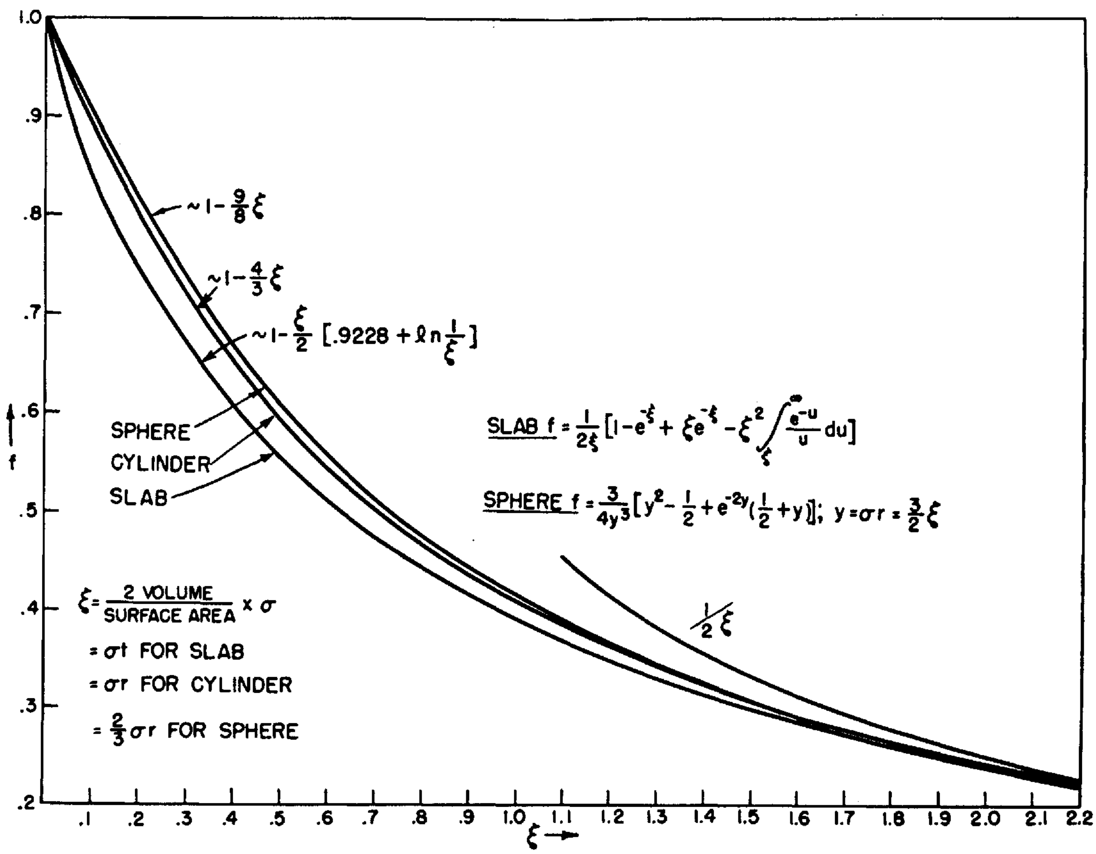

FIG. 1. Self-shielding factor for sphere, cylinder, and slab.

Consider the individual factors in Eq. (3). Since we are treating the case in which there are either one or zero lumps in each ΔS_i, it is possible to write

$$\langle e^{-\sigma l_i}\rangle = (1 - n\pi a^2 \Delta S_i) + n\pi a^2 \Delta S_i (1 - \tfrac{4}{3}\, a\sigma_0 f_0). \tag{5}$$

The first term on the right of Eq. (5) is the probability that there is no poison lump lying on the segment ΔS_i multiplied by the transmission probability for this case, namely unity. The second term is the product of the probability that a lump does lie on the segment ΔS_i by the average transmission probability, defined in terms of the self-shielding factor f_0 as $(1 - \frac{4}{3}\, a\sigma_0 f_0)$ where σ_0 is the (unshielded) macroscopic cross section of the poison in the lump.[1] Thus, writing V_0 for $\frac{4}{3}\,\pi a^3$, we have

$$\langle e^{-\sigma l_i}\rangle = 1 - n\Delta S_i V_0 \sigma_0 \tag{6}$$

and

$$\langle e^{-\sigma l}\rangle = \prod_i (1 - n\Delta S_i V_0 \sigma_0 f_0) \simeq \exp\,(-S_n V_0 \sigma_0 f_0) \tag{7}$$

where $S = \sum_i \Delta S_i$. Equation (7) thus states that a neutron beam in passing through the

[1] For more general shapes of the individual lumps, this factor is $1 - 2\xi f_0(\xi)$ where $f_0(\xi)$ is the self-shielding factor for the lump, and ξ is defined as $2V\sigma_0/S$ where V is the volume and S is the surface area of the lump. A graph of the self-shielding factor f as a function of ξ, obtained from reference 1, is included here for convenience. See Fig. 1.

mixture along a path length S is attenuated with a mean free path of $(nV_0\sigma_0 f_0)^{-1}$. Thus, the quantity $nV_0\sigma_0 f_0$ replaces the macroscopic cross section, where it should be pointed out again that f_0 is the self-shielding factor for the individual particles of poison (which may be referred to as a microscopic self-shielding factor).

In calculating the macroscopic self-shielding factor f_{mac} for the mixture, the usual expression for the absorption probability

$$P_A \propto V_{\text{mac}}\sigma_0 f_{\text{mac}}(\xi), \tag{8a}$$

is replaced by

$$P_A \propto V_{\text{mac}} n V_0 \sigma_0 f_0 f_{\text{mac}}(\xi). \tag{8b}$$

Thus the over-all self-shielding factor is

$$f_{\text{Tot}} = f_0(\xi_0) f_{\text{mac}}(\xi_{\text{mac}}), \tag{9}$$

with

$$\xi_0 = 2V_0\sigma_0/S_0\,, \tag{10a}$$

$$\xi_{\text{mac}} = \frac{2V_{\text{mac}}\, n\sigma_0\, V_0 f_0(\xi_0)}{S_{\text{mac}}}\,. \tag{10b}$$

Here V_{mac} and S_{mac} refer to the volume and surface area of the mixture.

In order to obtain a feeling for the magnitude of this effect, consider the self-shielding of a sphere of radius 1 cm filled with lumps of natural boron whose radius is 0.25 mm. Assume two per cent of the volume of the large sphere to be occupied by the boron. Then, we find $\xi_0 = 1.78$, $f_0(\xi_0) = 0.29$ leading to $\xi_{\text{mac}} = 0.41$, $f_{\text{mac}} = 0.67$, and thus $f_{\text{Tot}} = 0.19$. On the other hand, if the boron were homogeneously distributed throughout the large sphere, we would have $\xi_{\text{mac}} = 1.43$ and $f_{\text{mac}} = 0.32$. In general, the lumping of poison tends to increase the self-shielding.

REFERENCE

1. DWORK *et al.*, KAPL-1262. "Self-Shielding Factors for Infinitely Long, Hollow Cylinders."

Knolls Atomic Power Laboratory[2]
Schenectady, New York

H. HURWITZ, JR.
P. F. ZWEIFEL

Received July 2, 1956

[2] Operated by the General Electric Company for the U. S. Atomic Energy Commission.