The weighting coefficients would then be

$$(w_i)_{\text{econ}} = \frac{P_i}{P_9} \quad ,$$

where P_i is the price for isotope i. Such prices P_i depend on the evolution of the world market and not on the kind of reactor we are considering. The practical interest of such a definition is, however, questionable as most of such prices are not fixed presently.

P. Goldschmidt

Belgonucleaire
Rue des Colonies, 35
1000 Bruxelles, Belgium

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REPLY TO GOLDSCHMIDT

Dear Sir:

At the American Nuclear Society Meeting in San Francisco, 1969, I presented the paper[1] "An Improved Definition of the Breeding Ratio for Fast Reactors." The improvement consisted of a set of isotopic weight factors which reduce the sensitivity of the breeding gain to the composition of the fuel. The use of this improved definition avoids that an inaccuracy in the estimation of the equilibrium composition causes a large inaccuracy in the equilibrium breeding gain. The printed summary, limited to 450 words in addition to a figure, has been misinterpreted by Goldschmidt.[2] I present in this reply the more complete argumentation.

Goldschmidt states that the British definition of the breeding gain with weight factors w_i, representing the reactivity worths of isotope i, "is precisely the one that should be used to calculate the reactor doubling time." He states, in addition, that together with this definition one must "use the best average equilibrium composition." Goldschmidt's argumentation consists of the two parts which are the original basis of the British definition[3] of the breeding gain.

F1: "The doubling time of a given reactor is the time needed to build up an amount of fuel material sufficient to make another similar reactor critical."

F2: The isotope contribution to the critical mass is proportional to the reactivity worth of isotope i. This reactivity worth may be approximately calculated by average one group quantities.

The first point is obvious. The second point is long known. I used it, like others, in 1961 to calculate the contribution of the different plutonium isotopes to k_{eff}. On the basis of these two points Goldschmidt obviously concludes that "precisely the relative reactivity worths" should be used as weight factors in the calculation of the doubling time. In addition, Goldschmidt states by referring to his Eq. (7) that just the "fission rate" should be used as denominator in the breeding gain. In the first part of the following argumentation it is shown that one cannot conclude from the two facts, F1 and F2, which weight factors to use for the calculation of the equilibrium doubling time and also not which denominator to use in the definition of the breeding gain.

The first two questions I would like to discuss deal with the equilibrium fuel cycle which is the main subject of Goldschmidt's letter:

Q1: Is the use of the reactivity worths *necessary* for the calculation of the correct doubling time in an equilibrium state?

Q2: Is it *necessary* to use the fission rate as denominator of the breeding gain? [Comp. Eq. (7) in Ref. 2.]

DISCUSSION OF THE EQUILIBRIUM DOUBLING TIME (to Q1)

The equilibrium doubling time is commonly understood as the inverse "time constant" in an exponential accumulation of fuel. Depending on the average time Δt between the breeding events and the actual reuse of the bred fuel one defines the doubling time in two extreme cases:

t_{OD}, the doubling time without reuse

t_{CD} (= $\ln 2 \cdot t_{\text{OD}}$) the doubling time with instantaneous reuse, the "compound doubling time."

The actual doubling time (t_D) has a value between these two extremes depending on the time Δt. The questions to be discussed here are independent of this problem. We may therefore just use t_D without specifying its relation to t_{OD} and Δt.

Let $\hat{M}_f(t)$ be the accumulated "fuel" mass. The definition of "fuel" will be discussed below. $\hat{M}_f(t)$ is related to the doubling time by

$$\frac{\ln 2}{t_D} = \frac{1}{\hat{M}_f} \frac{d\hat{M}_f}{dt} \quad . \tag{1}$$

This leads to the exponential accumulation:

$$\hat{M}_f(t) = \hat{M}_f(o) \cdot \exp\left(\frac{\ln 2}{t_D} \cdot t\right) \quad . \tag{2}$$

For equilibrium operation the relative change of accumulated fuel is independent of the specific value of $M_f(t)$. Therefore, the right-hand side of Eq. (1) may be replaced by

$$\frac{1}{\hat{M}_f(t)} \cdot \frac{d\hat{M}_f(t)}{dt} = \frac{1}{M_f} \cdot \left(\frac{dM_f}{dt}\right) \quad , \tag{3}$$

where M_f is a certain fraction or multiple of $\hat{M}_f$, equal to the critical mass in a system fed with the accumulated fuel. Therefore, M_f has the same isotopic composition as $\hat{M}_f$. The right-hand side of Eq. (3) may then be used to calculate the doubling time:

$$\frac{\ln 2}{t_D} = \left[\frac{1}{M_f}\left(\frac{dM_f}{dt}\right)\right]^{eq} , \quad (4)$$

where the superscript, eq, is applied to indicate equilibrium quantities. For convertion of Eq. (4) into a corresponding one for the "number" of fuel nuclei, one can use the mass of a ^{239}Pu nucleus (m_9) as a close approximation for the masses of the higher plutonium isotopes:

$$N_f \simeq \frac{M_f}{m_9} = \frac{A_0}{239} \cdot M_f \quad (5)$$

(A_0 being the Avogadro number). This gives

$$\frac{\ln 2}{t_D} = \frac{1}{N_f^{eq}} \cdot \left(\frac{dN_f}{dt}\right)^{eq} . \quad (6)$$

The rate for surplus production of "fuel" is given by (using Goldschmidt's notations):

$$\left(\frac{dN_f}{dt}\right)^{eq} = \sum_i \gamma \ (C_{i-1} - A_i)^{eq} . \quad (7)$$

In the equilibrium state, the fuel for the "doubled" core is accumulated from equilibrium surplus breeding (S_i^{eq}). It has therefore the same isotopic composition as

$$S_i^{eq} = (C_{i-1} - A_i)^{eq} . \quad (8)$$

In other words the isotopic vector (N_f) of the accumulated "fuel" is proportional to S:

$$N_f = \begin{pmatrix} N_9 \\ \vdots \\ N_2 \end{pmatrix} = K \begin{pmatrix} S_9 \\ \vdots \\ S_2 \end{pmatrix} . \quad (9)$$

Inserting this into Eq. (6) gives

$$\frac{\ln 2}{t_D} = \frac{\sum_i \gamma_i S_i^{eq}}{\sum_i \gamma_i N_i^{eq}} = \frac{1}{K} , \quad (10)$$

independent of the choice of weight factors γ_i. This implies that all sets of weight factors for the plutonium isotopes give the same value for the equilibrium doubling time. Therefore, the answer to the first question posed above is:

A1: The reactivity worth weights are *not necessary* for the calculation of the equilibrium doubling time. Any set of weight factors (γ_i) will give the same result:

$$\frac{\ln 2}{t_D} = \left[\frac{\sum_i \gamma_i (C_i - A_i)}{\sum_i \gamma_i N_i}\right]^{eq} \text{ independent of } \gamma_i . \quad (11)$$

If, for example, the relative rate of fuel accumulation is 10% per year then the chemical element plutonium is accumulated with 10% per year as well as the thermally fissile component or any other combination of plutonium isotopes. This implies that the description of equilibrium breeding does not require a specific definition of "fuel."

The fact that the equilibrium doubling time is independent of the weight factors can be utilized for computational advantages. A very useful set of weight factors is the one which makes the right-hand side of Eq. (11) insensitive to inaccuracies in an estimate of the equilibrium fuel composition to be used in Eq. (11).

DEFINITIONS OF BREEDING GAIN (BG) AND BREEDING RATIO (BR) (to Q2)

A breeding gain (BG) may be split from the definition of the doubling time by division and multiplication with another reaction rate.

In the *conventional* form one relates the surplus production of fuel to the consumption of fuel:

$$\frac{\ln 2}{t_D} = \left(\frac{\sum_i \gamma_i A_i}{\sum \gamma N_i}\right)^{eq} \cdot \left[\frac{\sum_i \gamma_i (C_{i-1} - A_i)}{\sum_i \gamma_i A_i}\right]^{eq}$$

$$= \left(\frac{\sum_i \gamma A_i}{\sum_i \gamma_i N_i}\right)^{eq} \cdot BG_{conv}^{eq} . \quad (12)$$

The *British definition* relates the surplus production to the total unweighted fission rate:

$$\frac{\ln 2}{t_D} = \left(\frac{\sum_i F_i}{\sum_i w_i N_i}\right) \left[\frac{\sum_i w_i (C_{i-1} - A_i)}{\sum_i F_i}\right]^{eq}$$

$$= \left(\frac{\sum_i F_i}{\sum_i w_i N_i}\right)^{eq} \cdot BG_{UK}^{eq} . \quad (13)$$

A2: The *denominator* in the formula for the breeding gain *is arbitrary*. Each choice gives a breeding gain. The conventional choice of relating the surplus production rate of fuel to the consumption rate of the same type of fuel is more concrete and descriptive than the British definition.

The conventional definition allows also to introduce a breeding ratio to describe the complete balance:

$$BR^{eq} = BG^{eq} + 1 . \quad (14)$$

The introduction of the breeding gain separates the two basic phenomena which influence the multiplication of fuel:

1. The "breeding gain" contains the physics information about the relative surplus breeding.

2. The "first factor" contains the thermohydrolic information in form of the specific power.

In devising and optimizing breeder reactors *both* quantities, t_D and BG, are considered; e.g., the influence of certain design or cross section features on the BG is investigated independent of the eventual specific power.

ON THE DESCRIPTION OF NONEQUILIBRIUM BREEDING

In the equilibrium state the difference between the rates of production (C_{i-1}) and consumption (A_i) of the isotope "i" directly gives the rate of surplus production of isotope i. For nonequilibrium operation, however, the difference $C_{i-1} - A_i$ is inevitably used for the modification of the residing composition (to bring it closer to equilibrium). Only the rest is available as surplus production. In terms of a reactor integrated fuel cycle model (comp. Refs. 4 and 5) one has:

$$C_{i-1} - A_i - \frac{dN_i}{dt} = S_i(t) \qquad (15)$$

with $S_i(t)$ denoting the time-dependent surplus production rate of isotope i.

$N_i(t)$ and $S_i(t)$ can only be found by solving the complete fuel cycle problem consisting of the full set of equations (15) for all fuel isotopes, subject to constraint conditions (critically and fabricated fuel density) and completed by instructions on fuel management, in-core and out-of-pile. As a result one obtains the complete information about the time-dependent accumulation of each isotope i. The relative rate of accumulation is different for each isotope:

$$\frac{1}{N_i(t)} \frac{dN_i(t)}{dt} \text{ depends on } i. \qquad (16)$$

In the equilibrium this relative accumulation rate was the same for all i so that the fuel multiplication could be described by a single constant (k_0). Therefore, any combination of the plutonium isotopes was also multiplied by the same relative rate. For off-equilibrium operation a time-dependent relative rate of accumulation may be defined by condensing the detailed information with respect to various points of view:

$$\frac{\ln 2}{t_\gamma(t)} \approx \lambda_\gamma(t) = \frac{\sum_i \gamma_i S(t)}{\sum_i \gamma_i N_i(t)} , \qquad (17)$$

with $t_\gamma(t)$ and $\lambda_\gamma(t)$ defined by Eq. (17). For the accumulation of reactivity or of critical mass material certainly the isotopic "reactivity" worths must be used. In the same way for the accumulation of the fuel's monetary value the isotopic "monetary" worths have to be used. Other useful quantities to be calculated by condensation may be the relative accumulation rates of

the chemical element Pu ($\gamma_i = 1$ for all i)

the thermally fissile plutonium ($\gamma_i = 1$ for ^{239}Pu, ^{241}Pu and $= 0$ for ^{240}Pu, ^{242}Pu).

In summary: the nonequilibrium relative accumulation rate Eq. (17) describes different quantities for different sets of weight factors.

THE USE OF STATIC CALCULATIONS FOR THE ESTIMATIONS OF THE BREEDING GAIN

After the equilibrium composition of the surplus breeding has been found, one can calculate the BG from static calculations. During design and optimization of breeder reactor cores, neutron reaction cross sections and core design quantities are subject to frequent changes. Static one- and two-dimensional multigroup calculations are performed to find the corresponding changes in physics design quantities. This gives the incremental changes of these quantities reasonably accurate, except the breeding gain. The statically calculated incremental changes of the breeding gain may be off by a factor of 2 or more (comp. Refs. 1 and 6). With the $\bar{\eta}$ weights, proposed in Ref. 1, the breeding gain is largely insensitive to errors in the equilibrium composition of the fuel. Therefore, it is not necessary to find the changed equilibirum composition after each change in the system. The incremental changes of the BG as calculated by the formula of Ref. 1 seem to be accurate by about 10%. This is sufficiently high accuracy for incremental changes in most practical cases. Equation (7) of Ref. 2, however, may yield incremental changes which are off by more than a factor of 2, when it is applied without a complete analysis of the equilibrium fuel cycle for the modified system. Therefore, the $\bar{\eta}$ weighting of Ref. 1 represents an improvement for calculations of BG and of incremental changes of the BG from static calculation.

A similar insensitivity can be achieved for the doubling time. But this requires another set of weight factors, specifically devised to make Eq. (11) insensitive to variations in the composition.

K. O. Ott

Department of Nuclear Engineering
Purdue University
Lafayette, Indiana 47907

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