For the odd P_N approximation we obtain exactly $\frac{1}{2}(N+1)$ zeros $y^P_{N,m}$ by setting R_N equal to zero, as has been proved by Davison for the isotropic scattering. With our restrictions imposed upon the scattering law, Eq. (2), the proof remains unchanged in the case of anisotropic scattering. Thus, due to the ordering of the zeros for different orders of approximations,

$$y_0 < \ldots < y^P_{N+1,0} < y^P_{N,0} < y^P_{N-1,0} < \ldots . \quad (15)$$

and the properties of $R_n(y)$ functions

$$R_n(0) > 0 \quad \text{and} \quad \frac{d}{dy} R_n(y) < 0 \quad , \quad (16)$$

we conclude that the finite approximation dispersion function, Eq. (11), expressed in terms of R_{N-1} function

$$R_{N-1}(y) = y\, T(\nu_0) \quad (17)$$

has no zeros between 0 and $y_0 = 1/\nu_0^2$, because the value of $T(\nu_0)$ given by

$$T(\nu_0) = N\nu_0 \mathcal{Q}_N(\nu_0)/\mathcal{Q}_{N-1}(\nu_0) \quad (18)$$

is positive, since it is an even function of ν_0. Thus, y_0 is the smallest zero of Eq. (17) and the first part is proved.

Observing that even-order functions R_{2n} have positive finite limits as y goes toward infinity, that they have negative derivatives, and that they have poles at zeros of the corresponding R_{2n-1} functions, we conclude, by repeating the arguments of Davison, that Eq. (17) has exactly $\frac{1}{2}(N+1)$ distinct zeros for N odd, including y_0. This concludes the proof. As an example, Fig. 1 illustrates the relationship among zeros for P_5 and A_5 approximations.

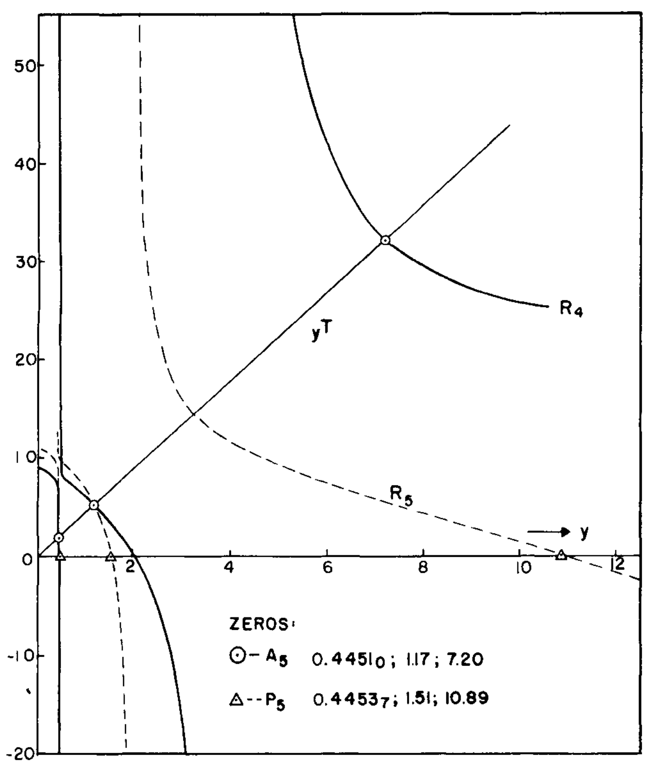

Fig. 1. Relative positions of zeros in P_5 and A_5 approximations for the scattering law with $f_0 = 0.75$; $f_1 = 0.25$; $f_2 = 0.10$; $f_3 = 0.05$; and $f_4 = 0.01$. If the interval around $y = 0.445$ is expanded, one obtains the picture of the first zeros that looks very similar to the situation around $y = 1.5$, where the second zeros are situated.

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Comments on an Article by J. Nilsson and R. Sandlin

In a recent paper, Nilsson and Sandlin[1] have discussed use of a source separation technique for investigating the attenuation of neutrons in annular ducts. In methods of this type, one attempts to separate the neutron flux into its several components by use of thin cadmium sheets. Of course, it is only thermal (subcadmium) neutrons that can be successfully separated in this manner. However, such techniques are potentially extremely valuable for obtaining a better understanding of the behavior of radiation in shields containing ducts. It is unfortunate that the work reported by Nilsson and Sandlin does not take full advantage of the information available in a source separation experiment.

The first problem encountered in this paper is concerned with the experimental techniques used. The authors have claimed that the measurements "account only roughly for the components as defined. . . ." However, the quantities which were measured do not even "roughly" correspond to the definitions of the various flux components. For instance, the streaming component (S) is supposed to include only those neutrons which enter the duct mouth in the thermal energy range and travel to the detector without striking the duct walls. However, the quantity that was measured includes all neutrons which enter the duct mouth in the thermal energy range. The true streaming component could have been measured by lining both walls of the annular duct with cadmium.

In addition, the albedo component (A) is supposed to include neutrons of all energies which enter the duct mouth and strike the walls before returning to the duct as thermal neutrons. The experimental arrangements used in the measurement of the albedo component contained cadmium covers at the duct mouth. Therefore, thermal neutrons were prevented from entering the duct mouth during this part of the experiment.

The second major difficulty encountered is the large uncertainty associated with some of the data. Use of data not known to better than a factor of 10 makes meaningful comparison with theory nearly impossible. For the source separation technique to be of maximum value, it is necessary to measure each flux component with reasonable accuracy. This can be accomplished by maintaining sufficiently high count rates during the measurement of basic quantities so that even the derived flux components have a small associated standard deviation.

The third problem encountered in this paper is the multiplying constants which appear with the theoretical expressions for each flux component. The constants that were found to give best agreement can be expected to apply

[1] J. NILSSON and R. SANDLIN, "Measured and Predicted Thermal- and Fast-Neutron Fluxes in Air-Filled Annular Ducts," *Nucl. Sci. Eng.*, 23, 3, 224 (1965).

only to the particular shield configuration investigated. It is interesting to observe whether or not theoretical expressions can be found that are approximately proportional to the data. However, this approach does not permit one to calculate the radiation attenuation for a different shield and duct arrangement.

In summary, the paper by Nilsson and Sandlin represents a disappointing attempt to use source-separation duct analysis for the following reasons:

1) Measured quantities did not closely approximate defined quantities.

2) The experimental uncertainty associated with some of the separated flux component data was too great to be useful in detailed theoretical analysis.

3) Theoretical correlations include undetermined multiplying constants.

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An Acknowledgment

In two recent papers I have discussed a method of modifying the one-group spherical harmonics method through the introduction of a flexible truncation scheme. In the first paper[1] only slab geometry was considered, and three examples of the truncation scheme were given, one of which led to a set of P-N-like equations with the property that for all $N \geqslant 1$ one pair of the exponential solutions in a homogeneous medium has the exact asymptotic transport-theory exponent. In a subsequent paper[2] this asymptotic truncation procedure was extended to an arbitrary, three-dimensional geometry.

Recently, I came across a set of lecture notes by B. Davison, dated 1947, in which the same asymptotic truncation procedure was suggested[3]. While Davison only considered homogeneous systems in slab geometry, and did not discuss the associated vacuum boundary conditions or the interface boundary conditions in a multilayered system, it must be acknowledged that the basic asymptotic truncation idea was suggested by Davison almost 20 years before my papers on the subject were written.

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[1]G. C. POMRANING, "A Generalized P-N Approximation for Neutron Transport Problems," *Nucleonik*, **6**, 348 (1965).

[2]G. C. POMRANING, "An Asymptotically Correct Approximation to the Multidimensional Transport Equation," *Nucl. Sci. Eng.*, **22**, 328 (1965).

[3]B. DAVISON, *Transport Theory of Neutrons*, Report LT-18, 124-129, National Research Council of Canada (1947).