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Letters to the Editors

Effect of Bragg Cutoff on the Diffusion of Thermal Neutrons in Graphite

INTRODUCTION

Sanatani and Kothari[1] have studied the diffusion of thermal neutrons in beryllium slabs of finite sizes by dividing the entire neutron energy range into two groups, one above and the other below the Bragg cutoff energy for this metal. Jain and Lawande[2] extended this study to the case of a semi-infinite slab of beryllium. They found that in beryllium at low temperature the asymptotic value (i.e. at distances from source plane tending to infinity) of the ratio of ϕ^1, the total neutron flux below the Bragg cutoff, to ϕ^2, the total neutron flux above the Bragg cutoff, is different from the value for a Maxwellian distribution at the temperature of the moderator. (Jain[3] has studied the same problem by dividing the entire energy range into 14 groups. He finds that in beryllium for $T = 300°$K the 2-group calculations and the 14-group calculations give approximately the same asymptotic value of ϕ^1/ϕ^2 whereas at $T = 100°$K he gets a little lower value of ϕ^1/ϕ^2.)

We here report some results of a similar study made in the case of a semi-infinite slab of graphite with an infinite plane source of neutrons at the free end of the slab. We have also studied the effect of change of absorption of the medium on the asymptotic value of ϕ^1/ϕ^2. The energy range of the first group is taken to be 0 to 22 k_0 (energy corresponding to the Bragg cutoff in graphite) and that of the second group to be 22 k_0 to 2000 k_0, where k_0 is the Boltzmann constant. We have chosen the upper limit of the thermal group to be 2000 k_0, since at this energy the Maxwellian tail becomes rather small. Following Kothari and Singwi[4] the scattering kernel has been calculated in the incoherent approximation using the model of lattice vibrations in graphite proposed by Krumhansl and Brooks[5]. In the energy region above the Bragg cutoff, the contribution from elastic scattering processes to the transport cross section, $\sigma_{tr}(E)$, has been calculated using Eq. (17) of Singwi and Kothari[6] for graphite temperatures of $T = 300°$K, 200°K and 150°K. Since in this energy region the inelastic scattering cross section does not exceed about 10% of the elastic scattering cross section, the contribution of inelastic processes to the average diffusion coefficient would be small and has been neglected. In the energy range below the Bragg cutoff we have taken $\sigma_{tr}(E)$ to be equal to $\sigma_1(E)$, where $\sigma_1(E)$ is the one-phonon inelastic coherent scattering cross section[6,7]. Since we have neglected the contribution of inelastic processes, Eq. (17) of Singwi and Kothari[6] will be an underestimation for σ_{tr} in the high energy region. To correct for this we have assumed σ_{tr} to be given by its asymptotic value, $\sigma_s\left(1-\frac{2A}{3}\right)$, beyond neutron energy 527 k_0 where σ_s is the scattering cross section and A is the mass number. The calculations at 300°K were compared with the earlier results[6] and were found to be in agreement within a few percent. Details can be seen in Ref. 8. The energy dependence of the absorption cross section, $\sigma_a(E)$, is assumed to be $1/v$ ($v = \left(\frac{2E}{m}\right)^{1/2}$ is the velocity of neutrons having energy E, and m is the neutron mass).

TWO-GROUP DIFFUSION EQUATIONS

Following Sanatani and Kothari[1], the two-group equations can be written as

$$\left(D^1 \frac{d^2}{dx^2} - \Sigma_a^1 - g^{21}\right)\phi^1(x) + g^{12}\phi^2(x) = 0 \tag{1}$$

$$g^{21}\phi^1(x) + \left(D^2 \frac{d^2}{dx^2} - \Sigma_a^2 - g^{12}\right)\phi^2(x) = 0 , \tag{2}$$

where

$$D^i = \frac{A}{\phi_M^i}\int_{E_{i-1}}^{E_i} D(E)\, E\, e^{-E/k_0T}\, dE; \qquad i=1,2 \text{ and } E_0=0 \tag{3a}$$

$$\Sigma_a^i = \frac{A}{\phi_M^i}\int_{E_{i-1}}^{E_i} \Sigma_a(E)\, E\, e^{-E/k_0T}\, dE \tag{3b}$$

$$g^{ij} = \frac{A}{\phi_M^i}\int_{E_{i-1}}^{E_i} dE \int_{E_{j-1}}^{E_j} \Sigma(E' \to E)\, E'\, e^{-E'/k_0T}\, dE'; \qquad i,j=1,2 \tag{3c}$$

$$\phi_M^i = A\int_{E_{i-1}}^{E_i} E\, e^{-E/k_0T}\, dE \tag{3d}$$

and

$A = 1/k_0^2 T^2$

$D(E) = \frac{1}{3\Sigma_{tr}(E)}$ is the diffusion coefficient at energy E

$\Sigma_{tr}(E)$ and $\Sigma_a(E)$ are the macroscopic transport and absorption cross sections, respectively, for neutron of energy E

$\Sigma(E' \to E)$ is the macroscopic energy transfer cross section for a neutron of energy E' being scattered into energy E.

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In deriving the above Eqs. (1) to (3), it is assumed that $\phi(x,E)$ can be separated into space and energy parts. Further in calculating the average values of the various quantities in Eqs. (3), the energy distribution of neutrons is assumed to be a Maxwellian. It may be mentioned that the average values of the various quantities will not depend sensitively on the actual energy distribution so long as the distribution is not drastically different from a Maxwellian.

Values of nuclear constants used in Eqs. (1) and (2) have been calculated for three temperatures of graphite - 150°K, 200°K and 300°K - and these are given in Table I.

The two-group Eqs. (1) and (2) have to be solved taking proper boundary conditions into account. Since the medium extends from $x = 0$ to $x = \infty$, we can write[1]

$$\phi^1(x) = A\,e^{-B_1 x} + C\,e^{-B_2 x} \qquad (4)$$

$$\phi^2(x) = A'\,e^{-B_1 x} + C'\,e^{-B_2 x}\ , \qquad (5)$$

where the values of B_1^2, B_2^2, A'/A and C'/C depend only on the nuclear constants and can be readily calculated[1]. Their values are given in Table II.

Thus to evaluate $\phi^1(x)$ and $\phi^2(x)$ completely we have to know only two constants, A and C, for which we need two boundary conditions. We define the source strength by

$$S_i = \lim_{x\to 0}\left[\frac{\phi^i}{4} - \frac{D^i}{2}\,\frac{d\phi^i(x)}{dx}\right]\ , \qquad (6)$$

where S_1 and S_2 are the number of neutrons emitted per second per square centimeter in group 1 and group 2, respectively. Here we consider the following cases:

a) $S_1 = 1$, $S_2 = 1$

b) $S_1 = 1$, $S_2 = 0$

c) $S_1 = 0$, $S_2 = 1$

d) $S_1/S_2 = \phi^1_M/\phi^2_M$ or $S_1 = \phi^1_M/\phi^2_M$ and $S_2 = 1$.

TABLE I

Average Values of Nuclear Constants for Graphite Occurring in Eqs. (1) and (2)

Constants	T = 150°K	T = 200°K	T = 300°K
D^1 cm	51.8	31.5	15.3
D^2 cm	0.92	0.92	0.92
$\Sigma_a^1(\text{cm}^{-1})$	1.58285×10^{-3}	1.57886×10^{-3}	1.57490×10^{-3}
$\Sigma_a^2(\text{cm}^{-1})$	3.86929×10^{-4}	3.38309×10^{-4}	2.80119×10^{-4}
$g^{12}(\text{cm}^{-1})$	5.31899×10^{-5}	5.6265×10^{-5}	5.70781×10^{-5}
$g^{21}(\text{cm}^{-1})$	5.38682×10^{-3}	9.94138×10^{-3}	2.19273×10^{-2}

TABLE II

Values of Parameters Occurring in Eqs. (4) and (5)

Parameters	T = 150°K	T = 200°K	T = 300°K
$B_1^2(\text{cm}^{-2})$	4.95059×10^{-4}	5.39644×10^{-4}	1.60475×10^{-3}
$B_2^2(\text{cm}^{-2})$	1.17829×10^{-4}	2.54731×10^{-4}	2.94851×10^{-4}
A'/A	-351.275	-97.5619	-19.2483
C'/C	16.2392	62.0478	332.564

TABLE III

Values of Constants Occurring in Eqs. (4) and (5)

Constants	T = 150°K	T = 200°K	T = 300°K
A	-9.43402×10^{-3}	-1.81992×10^{-2}	-3.2007×10^{-3}
C	3.32292×10^{-2}	3.36395×10^{-2}	1.14666×10^{-2}
A'	3.31393	1.77555	6.16083×10^{-2}
C'	0.53962	2.08726	3.81337

TABLE IV

Asymptotic Values of ϕ^1/ϕ^2 and Values of $1/B_2$ as a Function of Absorption Cross Section, σ_a

σ_a barns/ C atom	T = 150°K (ϕ^1/ϕ^2) asymptotic	T = 150°K $1/B_2$ cm	T = 200°K (ϕ^1/ϕ^2) asymptotic	T = 200°K $1/B_2$ cm	T = 300°K (ϕ^1/ϕ^2) asymptotic	T = 300°K $1/B_2$ cm
			$(\times10^{-2})$		$(\times10^{-3})$	
0.004	6.16×10^{-2}	92	1.61	63	3.01	58
0.008	8.55×10^{-2}	73	3.52	49	3.43	41
0.010	1.50×10^{-1}	65	4.58	45	3.65	37
0.012	1.78×10^{-1}	60	5.63	42	3.87	34
Ratio for Maxwel-lian flux	9.86×10^{-3}		5.66×10^{-3}		2.59×10^{-3}	

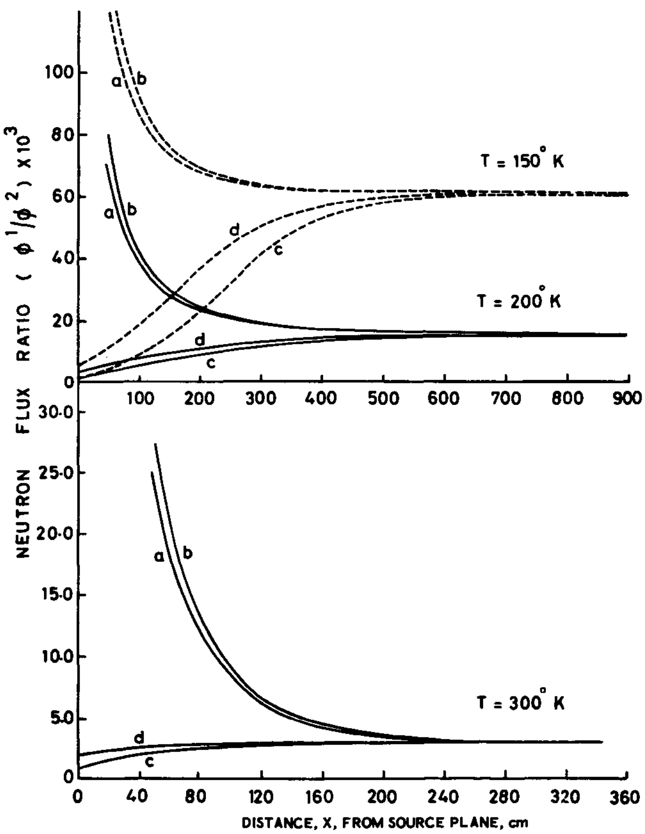

Fig. 1. The ratio of neutron fluxes in the two groups, ϕ^1/ϕ^2, plotted as a function of distance from the source plane for graphite temperatures of T = 300°K, 200°K and 150°K. At each temperature, the curves a), b), c) and d) represent the variation of ϕ^1/ϕ^2 with distance under different source conditions: Curve a): $S_1 = 1$, $S_2 = 1$; Curve b): $S_1 = 1$, $S_2 = 0$; Curve c): $S_1 = 0$, $S_2 = 1$; and Curve d); $S_1 = \phi_M^1/\phi_M^2$, $S_2 = 1$, where ϕ_M^1/ϕ_M^2 is the ratio for a Maxwellian flux and S_i is the number of neutrons emitted per second per unit area of the source plane in group i.

For the last case d), we give the values of A and C at the three temperatures in Table III.

In Table IV, we have given the asymptotic values of ϕ^1/ϕ^2 and the values of asymptotic diffusion length, $1/B_2$, for values of absorption cross section $\sigma_a = 0.004$, 0.008, 0.01 and 0.012 b at each temperature.

RESULTS AND DISCUSSION

In Fig. (1) we have plotted ϕ^1/ϕ^2 as a function of distance from the source plane for graphite temperature of $T = 300°K$, 200°K and 150°K, and for natural absorption ($\sigma_a = 0.004$ b). It will be seen that at each temperature in all cases a) to d), the ratio ϕ^1/ϕ^2 attains a certain constant value beyond a certain distance from the source plane. At $T = 300°K$, this constant value of the ratio ϕ^1/ϕ^2 is not much different from the Maxwellian value at $T = 300°K$, whereas at $T = 200°K$ and 100°K it deviates appreciably from the corresponding Maxwellian values. The effect of absorption on the asymptotic value of ϕ^1/ϕ^2 can be seen from Table IV. The asymptotic value of ϕ^1/ϕ^2 increases with the increase in the absorption of the medium, the increase being more at low temperatures.

The reason for the deviation of the asymptotic value of ϕ^1/ϕ^2 from the Maxwellian value at low temperatures lies in the fact that at low temperatures the transport mean free path of cold neutrons is much larger than that of thermal neutrons. This means that the thermal flux will drop off much faster (as one moves away from the source plane) than the cold flux, and consequently a large value of ϕ^1/ϕ^2 is obtained.

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Feroz Ahmed

Department of Physics and Astrophysics
University of Delhi,
Delhi-7, India.

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