in the two media. Although the expression (3) for σ_T is exact for a free gas, this is not to say that the representation of the flux by two overlapping groups is satisfactory.

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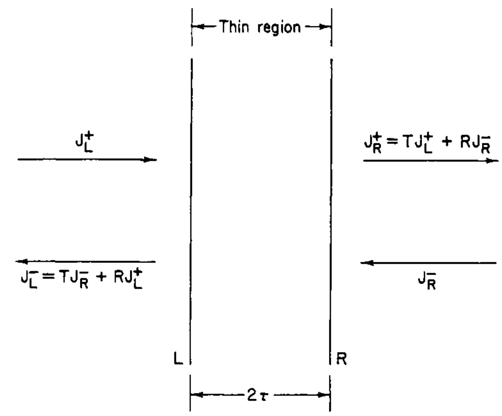

Fig. 1. Surface current relations for a symmetric thin region.

A Comment on the Treatment of Asymmetric Thin Regions*

Thin region theory (*1*) has been used extensively in reactor calculations for the treatment of geometrically thin, but often optically thick, regions. A typical assumption is that the transmitted, reflected, and incident neutron currents near a thin region of thickness $2t$, illustrated in Fig. 1, are approximately isotropic, and that the probabilities of transmission, T, and reflection, R, of neutrons incident on the region are independent of the direction (i.e., right or left) of incidence. The net currents on the left and right faces of the slab are then given by Eq. (7) of ref. *1* as[1]

$$\begin{aligned} J_L &= \frac{1}{2}\frac{1+T^2-R^2}{(1+R)^2-T^2}\varphi_L - \frac{T}{(1+R)^2-T^2}\varphi_R \\ J_R &= \frac{T}{(1+R)^2-T^2}\varphi_L - \frac{1}{2}\frac{1+T^2-R^2}{(1+R)^2-T^2}\varphi_R \end{aligned} \qquad (1)$$

where φ_L and φ_R are the fluxes at the left and right faces. While Eqs. (1) may be used directly as boundary conditions, a more frequently used form of the theory is based on the fact that the solution of the difference equations form of the diffusion equations for a region without interior mesh points yields expressions for the currents which are of the same form as Eq. (1); viz. (see Eq. (10) of ref. *1*)[1]

$$\begin{aligned} J_L &= \left(\frac{D}{2t} + \Sigma_a t\right)\varphi_L - \frac{D}{2t}\varphi_R \\ J_R &= \frac{D}{2t}\varphi_L - \left(\frac{D}{2t} + \Sigma_a t\right)\varphi_R \end{aligned} \qquad (2)$$

By equating coefficients of (1) and (2), a fictitious diffusion coefficient D and a fictitious absorption cross section Σ_a are obtained which allow the *formal* use of diffusion theory within the region, but which in fact preserve the transport theory relations (1) at the surfaces. Modifications of the procedure for the case in which interior mesh points are introduced in the region have also been developed (*2*, *3*).

Thin regions, however, are often introduced into inherently asymmetric configurations. A thin absorbing plate might, for example, be placed between a core and a reflector region in order to suppress a power peak. Then, because of the different spectral indices (*4*) of the materials to the left and right of the thin region, the energy averaged transmission and reflection probabilities for left incidence, T_L and R_L, might be different than the corresponding probabilities, T_R and R_R, for right incidence.[2] In such a case, it is easily shown that Eq. (1) should be replaced by

$$\begin{aligned} J_L &= \frac{1}{2}\frac{T_L T_R + (1-R_L)(1+R_R)}{(1+R_L)(1+R_R) - T_L T_R}\varphi_L - \frac{T_R}{(1+R_L)(1+R_R) - T_L T_R}\varphi_R \\ J_R &= \frac{T_L}{(1+R_L)(1+R_R) - T_L T_R}\varphi_L - \frac{1}{2}\frac{T_L T_R + (1+R_L)(1-R_R)}{(1+R_L)(1+R_R) - T_L T_R}\varphi_R \end{aligned} \qquad (3)$$

equations which contain four distinct coefficients, instead of the two which appear in (1).

It is tempting in this case to divide the slab into two regions with fictitious absorption and diffusion constants (Fig. 2), in the expectation that by comparing coefficients of the resulting integrated difference-diffusion equations with (3), the fictitious constants may be determined. However, the relations obtained by this strategem[3] are

$$\begin{aligned} J_L &= \left(\frac{D_1}{t} + \frac{\Sigma_1 t}{2} - \frac{2D_1^2/t}{E}\right)\varphi_L - \frac{2D_1 D_2/t}{E}\varphi_R \\ J_R &= \frac{2D_1 D_2/t}{E}\varphi_L - \left(\frac{D_2}{t} + \frac{\Sigma_2 t}{2} - \frac{2D_2^2/t}{E}\right)\varphi_R \\ E &= 2(D_1 + D_2) + (\Sigma_1 + \Sigma_2)t^2 \end{aligned} \qquad (4)$$

which contain, not the necessary four, but only three dis-

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* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] It is assumed here that the thin region is source free.

[2] If the partial current spectra on each side of the slab were the same, then the left and right transmissions would be the same. There is at present, however, no reason to believe the partial current spectra are not different.

[3] Equations (4) are obtained by integrating the difference-diffusion equation in the slab from the left face to

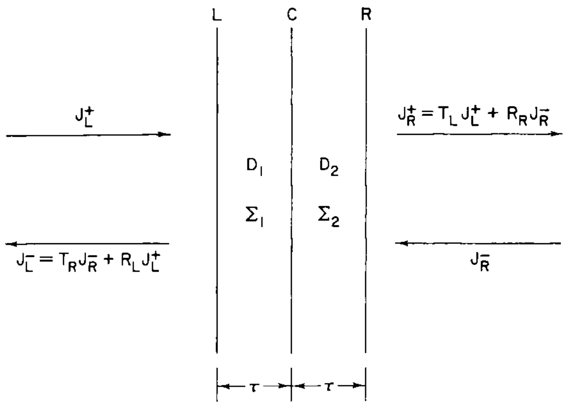

FIG. 2. Surface current relations for an asymmetric thin region.

tinct coefficients. Equations (4) have previously been obtained by G. H. Miley (*5*), who also considered the introduction of more than two regions, and of off-center internal mesh points, but always with the result that the relations between J_L, J_R, φ_L and φ_R involved only three distinct coefficients.

The purpose of this note is to point out the reason for the failure of this apparently straightforward approach. The key to the difficulty is found by inspection of Eq. (3). It will be recalled that the one velocity diffusion equation is self-adjoint (*6*) for both the differential and difference equation forms. The boundary conditions on the equation should also be self-adjoint if diffusion theory is to be used within the slab, but Eq. (3), which is the boundary condition to be applied on the opposite sides of the thin region, is self-adjoint only if $T_L = T_R$ (in which case Eq. (3) can in fact be equated to Eq. (4), and the required three (in this case) coefficients obtained).

The observation that the approach succeeds if $T_L = T_R$ leads to the physical origin of the difficulty. The basic reason for the failure of the scheme is that while T_L and T_R, which represent energy averaged quantities, may well be different, one is trying to represent the situation in a one-velocity theory, and a directionally transmitting slab does not exist in the monoenergetic case. That energy averaged (i.e., one-velocity) treatments of transmission probabilities necessarily predict equal left and right transmissions is a consequence of the reciprocity theorem for monoenergetic transport theory. The form of the theorem which is of immediate interest is given by Eq. (22) of ref. *7* as

$$|\mathbf{n}_R \cdot \mathbf{\Omega}_R| \, \varphi(R, -\mathbf{\Omega}_R; L, \mathbf{\Omega}_L) = |\mathbf{n}_L \cdot \mathbf{\Omega}_L| \, \varphi(L, -\mathbf{\Omega}_L; R, \mathbf{\Omega}_R) \tag{5}$$

which states that the *emergent* normal component of the angular flux in direction $-\mathbf{\Omega}_R$ at the right face of the slab due, to an *incident* beam at the left face in direction $\mathbf{\Omega}_L$, is equal to the emergent normal component of the angular flux in direction $-\mathbf{\Omega}_L$ at the left face due to an incident beam

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half way through the first region, from the latter point to the midpoint of region 2, and from this point to the right face. Three equations relating J_L, J_R, φ_L, φ_R, and φ_0 (the flux at the center line) are obtained. Elimination of φ_0 from these equations results in (4).

in direction $\mathbf{\Omega}_R$ at the right face. If it is now assumed that the neutron beams incident on the left and right faces are identical in their angular distributions (an assumption which is made in the derivation of Eqs. (1) snd (3)), then integration of (5) over $\mathbf{\Omega}_L$ and $\mathbf{\Omega}_R$ yields $T_L = T_R$. It follows that any treatment of thin regions that presupposes different left and right transmission probabilities cannot correspond to a one-velocity transport description of the system. Hence, Eqs. (3) and (4) should not be expected to be commensurate: one is trying to represent directional transmissions within the framework of a formalism which is inherently incapable of doing so.

To show more specifically that $T_L = T_R$ in monoenergetic *diffusion* theory, consider two problems. Let $\varphi_1(x)$ be the solution to the diffusion equation

$$-\frac{d}{dx} D(x) \frac{d\varphi_1(x)}{dx} + \Sigma(x)\varphi_1(x) = 0 \tag{6a}$$

subject to homogeneous boundary conditions on $\varphi_1(x)$ at the slab faces. Similarly, let $\varphi_2(x)$ be the solution to

$$-\frac{d}{dx} D(x) \frac{d\varphi_2(x)}{dx} + \Sigma(x)\varphi_2(x) = 0 \tag{6b}$$

subject to homogeneous boundary conditions on φ_2. Upon multiplying (6a) by φ_2, (6b) by φ_1, integrating over the slab volume, and subtracting, the result

$$\int_L^R \left(\varphi_2 \frac{d}{dx} D \frac{d\varphi_1}{dx} - \varphi_1 \frac{d}{dx} D \frac{d\varphi_2}{dx} \right) dx = 0 \tag{7}$$

is obtained. If each term is now integrated by parts once, Eq. (7) reduces to

$$\varphi_2(L)D(L)\frac{d\varphi_1(L)}{dx} - \varphi_2(R)D(R)\frac{d\varphi_1(R)}{dx} = \varphi_1(L)D(L)\frac{d\varphi_2(L)}{dx} - \varphi_1(R)D(R)\frac{d\varphi_2(R)}{dx}$$

an expression which can be rewritten in terms of the partial currents

$$J^{\pm} = \tfrac{1}{4}\varphi \mp \tfrac{1}{2} D \frac{d\varphi}{dx}$$

as

$$J_{2R}^+ J_{1R}^- - J_{2R}^- J_{1R}^+ = J_{1L}^- J_{2L}^+ - J_{1L}^+ J_{2L}^- \tag{8}$$

In the above equations, $D(L)$ is $D(x)$ evaluated at the left surface of the slab, etc.

Now, let φ_1 be the solution to the problem with unit incoming current in the left, and zero incoming current on the right;

$$J_{1L}^+ = 1, \quad J_{1R}^- = 0 \tag{9a}$$

Similarly, let φ_2 be the solution to the problem with unit incoming current on the right and zero incoming current on the left;

$$J_{2R}^- = 1, \quad J_{2L}^+ = 0 \tag{9b}$$

The insertion of (9a, b) into (8) gives at once

$$J^+_{1R} = J^-_{2L}$$

With the normalization of Eqs. (9a, b), however, the last expression is just the identity

$$T_L \equiv T_R$$

proving that no distribution of effective diffusion constants and absorption cross sections within the slab is capable of representing directional transmissions in monoenergetic diffusion theory.

The author wishes to acknowledge many conversations with G. H. Miley, whose work suggested the problem, and with P. B. Daitch, one of whose remarks led the author to the particular result reported herein. The comments of the reviewers, which were of considerable value in the organization of this note, are also acknowledged.

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Apparent Boiling of Uranium Oxide in the Center of a Fuel Pin During Transient Power Generation

Ceramic fuels, and particularly mixed uranium–plutonium oxide, are of considerable interest for use in a new concept of fast reactor termed the FCR[1] and extensive development work on fabrication and irradiation performance of these fuels is now in progress. This work, in common with that for many nuclear reactor concepts, includes a study of the performance of the fuels under severe transient power generation conditions such as may occur during an accidental nuclear excursion. During one of the more violent of these power excursion tests it appears that the UO_2 in a substantial portion of the test fuel pin reaches its boiling point. The mode of this boiling, and some of the occurrences preceding it, may have considerable significance to many reactor accident studies.

This series of tests of oxide fuels was undertaken with the assistance of ANL—the operators of the TREAT (Transient Reactor Test) facility, and had the following objectives:

Investigate the performance characteristics of this class of fuel when subjected to transients of varying severity; observations to include such aspects as thermal expansion of fuel, thermal stress effects on clad, materials interaction during transient high temperatures, damage or redistribution of fuel material, etc.

Determine the performance limit of this class of fuel when clad with stainless steel and employed in a sodium-cooled system; two cases of particular interest being the performance limit of the fuel for the reference power reactor, and of the fuel for the specially designed EFCR (Experimental Fast Ceramic Reactor), in which it is intended to carry out severe power excursions for analytical and demonstration purposes.

The plan of the experiments called for a sodium-filled capsule, designed as a calorimeter to permit computation of maximum fuel temperature and integrated power, and suitable for insertion in the TREAT facility in place of a standard fuel assembly. The capsules have instruments to

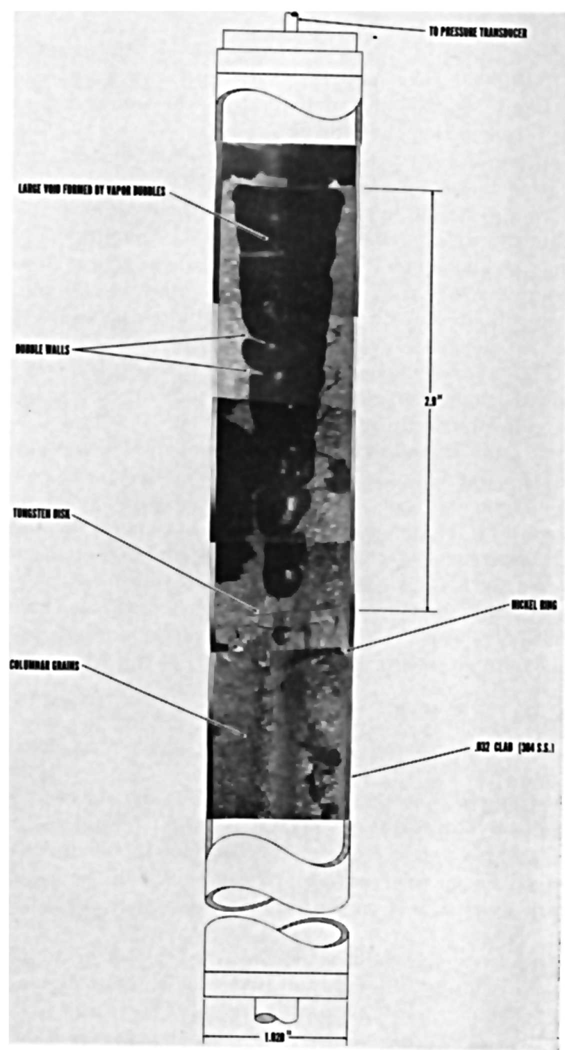

Fig. 1. Upper section APED-TREAT sample #10

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[1] Fast Ceramic Reactor, presently under development by General Electric in a program sponsored by the USAEC.