with this identification, expressions for resonance absorption will be the same in hetrogeneous and homogeneous systems.[1]

For example, in the NR approximation the expression for the resonance integral in a homogeneous system is

$$I(NR) = \int \sigma_a \frac{\sigma_m + \sigma_{sp}}{\sigma_m + \sigma_t} \frac{dE}{E}$$

where σ_{sp} is the potential scattering cross section of the fuel. With the equivalence theorem, one can immediately write for a heterogeneous system

$$I(NR) = \int \sigma_a \frac{\sigma_t \left(\dfrac{1 - P_0}{P_0}\right) + \sigma_{sp}}{\sigma_t \left(\dfrac{1 - P_0}{P_0}\right) + \sigma_t} \frac{dE}{E}$$

which with some use of algebra becomes identical with the expression commonly in use.

Note that the above work requires that the resonance be narrow with respect to neutron energy loss in moderator collisions but, except for the neglect of the spacial variation of the resonance flux in the fuel, the derivation is independent of the mechanics of energy loss in fuel collisions. Thus, with this approximation, the equivalence relationship is valid not only in the NR approximation but also in the NRIA case as well as in intermediate situations.

If the Wigner approximation

$$1 - P_0 = \frac{S}{4VN_0 \sigma_t + S}$$

is used, then $\sigma_t[(1 - P_0)/P_0]$ is constant and equal to $S/4N_0V$ where S is the surface area and V the volume of the lump. This is the case discussed in (*1*).

REFERENCES

1. J. Chernick and R. Vernon, *Nuclear Sci. and Eng.* **4**, 649 (1958).

2. K. M. Case, F. deHoffman, and G. Placzek, "Introduction to the Theory of Neutron Diffusion." U. S. Government Printing Office, Washington, D. C. (1955).

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[1] Note that $\sigma_t[(1 - P_0)/P_0]$ is energy dependent while σ_m is constant. Thus in using the equivalence relationship, care must be exercised that the substitution of

$$\sigma_t[(1 - P_0)/P_0]$$

for σ_m is made before integration or other operations involving the energy dependence of σ_m are performed.

The Effect of Bragg Cutoff on the Diffusion of Thermal Neutrons in a Semi-Infinite Slab of Beryllium

Sanatani and Kothari (*1*) have studied the diffusion of thermal neutrons grouped in the energy ranges below and above the Bragg cut-off in a finite sized beryllium slab, having an infinite plane source at one end of the slab. In each energy range the neutrons were assumed to have Maxwellian distribution corresponding to the temperature of the moderator. They made calculations for two temperatures, $T = 100°K$ and $T = 300°K$.

In their study, they considered a slab so small that the equilibrium spectrum characteristic of the temperature of the medium is not always established. Consequently, the effect of the transfer terms on the diffusion of neutrons is not clearly illustrated. Therefore, we extended the study to a semi-infinite slab of beryllium, i.e., the medium extending from $x = 0$ to $x = \infty$. The value of various nuclear constants averaged over the Maxwellian in their respective energy ranges are taken from ref. *1*. We studied the problem under various source conditions in the form of inward cold and thermal currents, neutrons/sec/cm², of strengths Q_1 and Q_2 respectively, at $x = 0$.

We studied the following cases:

(i) $Q_1 = 1$, $Q_2 = 1$.
(ii) $Q_1 = 1$, $Q_2 = 0$.
(iii) $Q_1 = 0$, $Q_2 = 1$.
(iv) Q_1 and Q_2 are taken such that at $x = 0$, ψ_1/ψ_2 = Maxwellian ratio at the temperature of the slab, where ψ_1 and ψ_2 are respectively the cold and thermal fluxes.

The case where Q_1/Q_2 is in the Maxwellian ratio is very close to case (iv) and its study will not provide any extra information. Moreover, this case has already been studied in ref. *1*.

All the above cases have been studied for two temperatures, $T = 100°K$ and $T = 300°K$. The results of the calculation are given in Fig. 1 and Fig. 2 corresponding to 300°K and 100°K respectively. In these figures we have plotted only the ratio ψ_1/ψ_2 as a function of x. We draw the following conclusions from this study.

We see from Figs. 1 and 2 that whatever be the source we feed in the system at $x = 0$, the quantity ψ_1/ψ_2 attains a certain equilibrium value in the asymptotic region. The equilibrium value is characteristic of the temperature of the medium. If the absorption cross section Σ_a of the

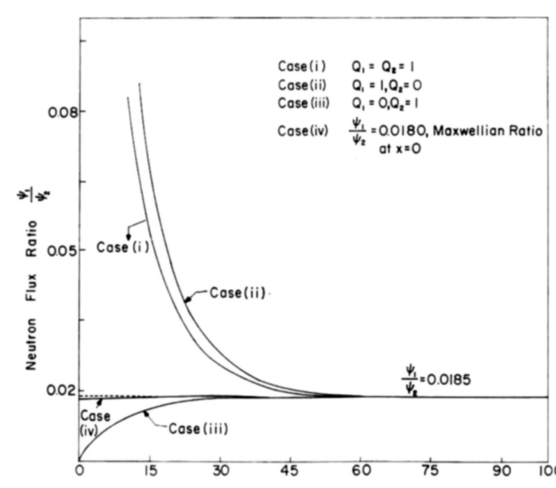

Fig. 1. Variation of neutron flux ratio, ψ_1/ψ_2, as a function of x under various source conditions at temperature $T = 300°K$.

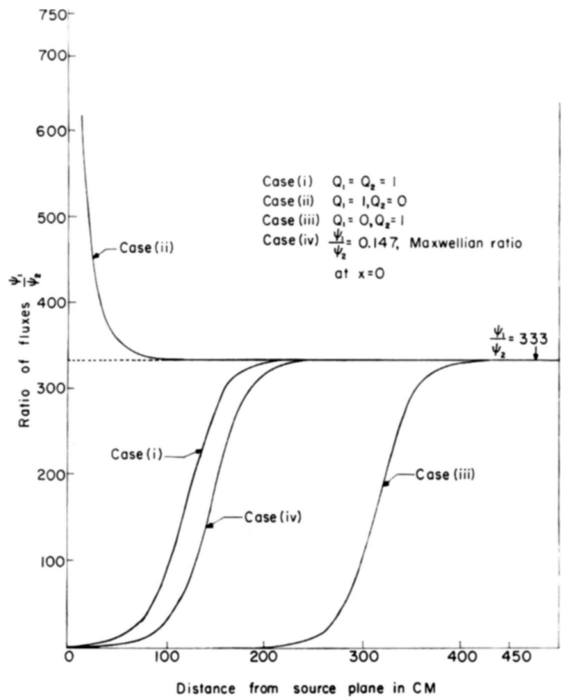

FIG. 2. Variation of neutron flux ratio, ψ_1/ψ_2, as a function of x under various source conditions at temperature $T = 100°K$.

medium were zero, then the equilibrium value of ψ_1/ψ_2 would always be Maxwellian corresponding to the temperature of the slab. This follows immediately from the principle of detailed balance.

In a practical case there is always some absorption in the medium, as in the case under consideration. Figure 1 shows that at $T = 300°K$, the equilibrium value $\psi_1/\psi_2 = 0.0185$ is not much different from the Maxwellian value, 0.0180. We cut off the Maxwellian at the higher energy region at the energy $E = 2000\ k_0$, where k_0 is Boltzmann's constant. In the case where Q_1/Q_2 is such that ψ_1/ψ_2 is Maxwellian at the origin this quantity remains practically constant over the whole slab.

In the case of $T = 100°K$, Fig. 2 shows that the value of ψ_1/ψ_2 in the asymptotic region is as high as approximately 2400 times the value of the Maxwellian ratio, 0.147. In this case, therefore, if we choose Q_1/Q_2 such that ψ_1/ψ_2 is in the Maxwellian ratio at the origin, then this ratio does not remain constant but increases steadily with x until the asymptotic region is reached.

The reason for this behavior at $T = 100°K$ lies in the fact that the values of κ^2, the inverse diffusion area, in the two energy groups are much larger than the values of interaction terms, as can be seen from ref. *1*. Consequently, the simple diffusion phenomenon is dominant over the transfer phenomenon. Therefore, the space variation of ψ_1/ψ_2 is mainly governed by the diffusion properties of the medium. Since, at $T = 100°K$, the diffusion length, $1/\kappa$, for the cold neutrons is much larger than that for the thermal neutrons; the thermal flux decays must faster than the cold flux. This explains why we obtain a very high value of ψ_1/ψ_2 in the asymptotic region. At $T = 300°K$, the values of the interaction terms are comparable with the varlues of κ^2, the inverse diffusion area, for the two groups of neutrons. Further, the diffusion lengths for the two groups of neutrons are not widely different. These conditions always tend to keep the Maxwellian ratio in the equilibrium region.

If we neglect the interaction terms completely, then the flux is governed by the two independent simple diffusion equations in their respective energy groups. Therefore the fluxes will decay according to the values of the diffusion length in the two energy groups. Since the diffusion length for the cold neutrons at either temperature of the medium is greater than that for the thermal neutrons, the decay of thermal neutrons is faster than the cold neutrons. This makes the ratio ψ_1/ψ_2 attain an infinite value as $x \to \infty$ at both temperatures.

In the region before the asymptotic region the variation of ψ_1 or ψ_2 and, hence, ψ_1/ψ_2 depends upon the source conditions. Their variations can be very large if the source conditions are such that value of ψ_1/ψ_2 at $x = 0$ is widely different from the asymptotic value. The interesting cases are when either Q_1 or Q_2 is equal to zero; then the corresponding flux will show the build-up. This can be understood from the curves for cases (ii) and (iii) in Figs. 1 and 2. If the source conditions are such that the value of ψ_1/ψ_2 at $x = 0$ is close to its value in the asymptotic region, then the effect of interaction terms is such that both ψ_1 and ψ_2 decay at the same rate and, therefore, ψ_1/ψ_2 remains constant for all values of x. This can be seen from the curve for case (iv) in Fig. 1 where ψ_1/ψ_2 at $x = 0$ is in Maxwellian ratio and its value is very close to the asymptotic value at $T = 300°K$.

Thus, we see from the above discussion that we cannot study the true nature of the interaction phenomenon in a small finite sized slab where the equilibrium is not established. The leakage of neutrons in this case further complicates the problem.

REFERENCE

1. S. SANATANI AND L. S. KOTHARI, *Nuclear Sci. and Eng.*, **11**, 211 (1961).

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