Letters to the Editors

Re: Solid State Electrolysis in Yttrium Metal

The suggestion of Williams and Huffine (*1*) on combining solid state electrolysis with zone refining coincides with the writer's observation in the May 1 issue of *Chem. Eng. News*, page 5.

Some years ago the writer suggested the combined method for zirconium purification but this aroused little interest. Recent success on beryllium purification by Drs. Herman and Wilsdorf at Franklin Institute prompted the writer to again suggest the combined technique. This time the response was one of immediate interest and I am pleased to report that the technique will now be tested.

During our correspondence Dr. Herman called my attention to the above article (*1*). My reply indicated the possible application also to the Czochralski technique. On closer reading of the article I note that this was apparently implied by the authors in their suggested application to single crystals of reactive metals.

REFERENCE

1. J. M. WILLIAMS AND C. L. HUFFINE, *Nuclear Sci. and Eng.* **9,** 500 (1961).

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A Generalized Equivalence Theorem For Resonance Escape in Heterogeneous Systems

Chernick and Vernon (*1*) have outlined an equivalence relationship between resonance absorption in heterogeneous and homogeneous systems. (According to them the suggestion of an equivalence was first advanced by Bakshi.) The development in (*1*) is based on the use of Wigner's rational approximation for the escape probability. In the following note it is shown that a general equivalence relationship can be written directly in terms of the escape probability.

Consider a fuel lump (or fuel lumps) of material with total cross section per fuel atom $\sigma_t(E)$, and scattering cross sections per fuel atom $\sigma_s(E)$. Let the fuel lump be embedded in a moderator of constant (scattering) cross section σ_{m1}. Then the average flux in the fuel lump ϕ_0 is given by the expression

$$V_0\,\phi_0(E)\sigma_t(E) = V_0\,P_0'(E)\int_E^{E/\alpha_0}\frac{\sigma_s(E)}{1-\alpha_0}\,\phi_0(E)\,\frac{dE}{E} + \frac{N_1}{N_0}\,V_1[1-P_1(E)]\int_E^{E/\alpha_1}\frac{\sigma_{m1}\,\phi_1(E)}{1-\alpha_1}\,\frac{dE}{E} \tag{1}$$

The zero subscript refers to events in the fuel lump and the subscript 1 refers to the moderator. It is assumed that the flux ϕ_1 in the moderator is uniform in space and that the scattered flux is isotropic. $P_0'(E)$ is the fraction of neutrons of energy E born within the fuel lump which have their next collision in the lump. $[1 - P_1(E)]$ is the probability that a neutron born in the moderator does not suffer its next collision in the moderator. N_1 is the moderator atom density and N_0 the atom density of the fuel.

In the narrow resonance approximation, the resonance width is assumed to be negligible compared to the energy spread between E and E/α_1. In this case, letting $\phi_1(E) = K/E$ the last term of (1) becomes

$$\frac{N_1V_1}{N_0}(1-P_1)\int_E^{E/\alpha_1}\frac{\sigma_{m1}\,\phi_1}{1-\alpha_1}\,\frac{dE}{E} = \frac{N_1}{N_0}\,\sigma_{m1}\,V_1(1-P_1)\,\frac{K}{E} \tag{2}$$

Now, as pointed out by Chernick, it is easily shown from (2) that $N_1\sigma_{m1}V_1(1 - P_1) = N_0\sigma_tV_0(1 - P_0)$; where $(1 - P_0)$ is the escape probability for a *uniform* isotropic source distribution in the fuel lump. Thus, if the assumption is made that $P_0' = P_0$ (1) can be written

$$\frac{\phi_0(E)\sigma_t(E)}{P_0(E)} = S_0(E) + \frac{K}{E}\,\frac{\sigma_t(E)[1-P_0(E)]}{P_0(E)} \tag{3}$$

or

$$\phi_0\left(\sigma_t + \sigma_t\,\frac{(1-P_0)}{P_0}\right) = S_0 + \frac{K}{E}\,\sigma_t\left(\frac{1-P_0}{P_0}\right) \tag{3a}$$

where

$$S_0(E) = \int_E^{E/\alpha_0}\frac{\sigma_s(E)}{1-\alpha_0}\,\phi_0(E)\,\frac{dE}{E}$$

In the homogeneous case the corresponding equation is

$$\phi(\sigma_t + \sigma_m) = S_0 + \frac{K}{E}\,\sigma_m \tag{4}$$

where σ_m is the moderator cross section per fuel atom. Comparison of (3a) and (4) shows that the expressions are formally identical if $\sigma_T[(1 - P_0)/P_0]$ is identified with σ_m. Thus, the generalized equivalence relationship states that with this identification, expressions for resonance absorption will be the same in hetrogeneous and homogeneous systems.[1]

For example, in the NR approximation the expression for the resonance integral in a homogeneous system is

$$I(NR) = \int \sigma_a \frac{\sigma_m + \sigma_{sp}}{\sigma_m + \sigma_t} \frac{dE}{E}$$

where σ_{sp} is the potential scattering cross section of the fuel. With the equivalence theorem, one can immediately write for a heterogeneous system

$$I(NR) = \int \sigma_a \frac{\sigma_t \left(\frac{1 - P_0}{P_0}\right) + \sigma_{sp}}{\sigma_t \left(\frac{1 - P_0}{P_0}\right) + \sigma_t} \frac{dE}{E}$$

which with some use of algebra becomes identical with the expression commonly in use.

Note that the above work requires that the resonance be narrow with respect to neutron energy loss in moderator collisions but, except for the neglect of the spacial variation of the resonance flux in the fuel, the derivation is independent of the mechanics of energy loss in fuel collisions. Thus, with this approximation, the equivalence relationship is valid not only in the NR approximation but also in the NRIA case as well as in intermediate situations.

If the Wigner approximation

$$1 - P_0 = \frac{S}{4VN_0 \sigma_t + S}$$

is used, then $\sigma_t[(1 - P_0)/P_0]$ is constant and equal to $S/4N_0V$ where S is the surface area and V the volume of the lump. This is the case discussed in *(1)*.

REFERENCES

1. J. Chernick and R. Vernon, *Nuclear Sci. and Eng.* **4,** 649 (1958).

2. K. M. Case, F. deHoffman, and G. Placzek, "Introduction to the Theory of Neutron Diffusion." U. S. Government Printing Office, Washington, D. C. (1955).

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[1] Note that $\sigma_t[(1 - P_0)/P_0]$ is energy dependent while σ_m is constant. Thus in using the equivalence relationship, care must be exercised that the substitution of

$$\sigma_t[(1 - P_0)/P_0]$$

for σ_m is made before integration or other operations involving the energy dependence of σ_m are performed.

The Effect of Bragg Cutoff on the Diffusion of Thermal Neutrons in a Semi-Infinite Slab of Beryllium

Sanatani and Kothari *(1)* have studied the diffusion of thermal neutrons grouped in the energy ranges below and above the Bragg cut-off in a finite sized beryllium slab, having an infinite plane source at one end of the slab. In each energy range the neutrons were assumed to have Maxwellian distribution corresponding to the temperature of the moderator. They made calculations for two temperatures, $T = 100°K$ and $T = 300°K$.

In their study, they considered a slab so small that the equilibrium spectrum characteristic of the temperature of the medium is not always established. Consequently, the effect of the transfer terms on the diffusion of neutrons is not clearly illustrated. Therefore, we extended the study to a semi-infinite slab of beryllium, i.e., the medium extending from $x = 0$ to $x = \infty$. The value of various nuclear constants averaged over the Maxwellian in their respective energy ranges are taken from ref. *1*. We studied the problem under various source conditions in the form of inward cold and thermal currents, neutrons/sec/cm², of strengths Q_1 and Q_2 respectively, at $x = 0$.

We studied the following cases:

(i) $Q_1 = 1$, $Q_2 = 1$.
(ii) $Q_1 = 1$, $Q_2 = 0$.
(iii) $Q_1 = 0$, $Q_2 = 1$.
(iv) Q_1 and Q_2 are taken such that at $x = 0$, ψ_1/ψ_2 = Maxwellian ratio at the temperature of the slab, where ψ_1 and ψ_2 are respectively the cold and thermal fluxes.

The case where Q_1/Q_2 is in the Maxwellian ratio is very close to case (iv) and its study will not provide any extra information. Moreover, this case has already been studied in ref. *1*.

All the above cases have been studied for two temperatures, $T = 100°K$ and $T = 300°K$. The results of the calculation are given in Fig. 1 and Fig. 2 corresponding to 300°K and 100°K respectively. In these figures we have plotted only the ratio ψ_1/ψ_2 as a function of x. We draw the following conclusions from this study.

We see from Figs. 1 and 2 that whatever be the source we feed in the system at $x = 0$, the quantity ψ_1/ψ_2 attains a certain equilibrium value in the asymptotic region. The equilibrium value is characteristic of the temperature of the medium. If the absorption cross section Σ_a of the

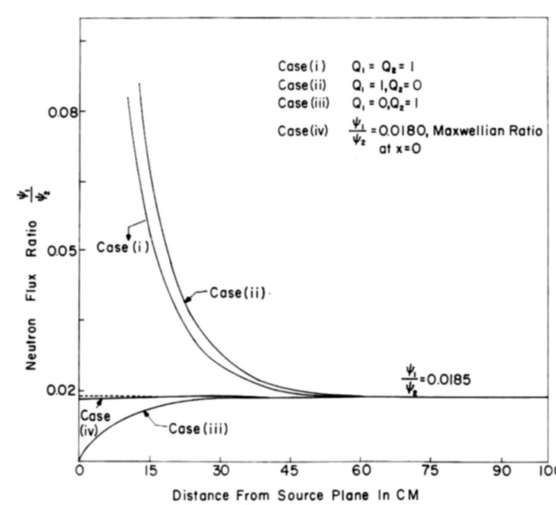

Fig. 1. Variation of neutron flux ratio, ψ_1/ψ_2, as a function of x under various source conditions at temperature $T = 300°K$.