# Letters to the Editors

## Re: Solid State Electrolysis in Yttrium Metal

The suggestion of Williams and Huffine (*1*) on combining solid state electrolysis with zone refining coincides with the writer's observation in the May 1 issue of *Chem. Eng. News*, page 5.

Some years ago the writer suggested the combined method for zirconium purification but this aroused little interest. Recent success on beryllium purification by Drs. Herman and Wilsdorf at Franklin Institute prompted the writer to again suggest the combined technique. This time the response was one of immediate interest and I am pleased to report that the technique will now be tested.

During our correspondence Dr. Herman called my attention to the above article (*1*). My reply indicated the possible application also to the Czochralski technique. On closer reading of the article I note that this was apparently implied by the authors in their suggested application to single crystals of reactive metals.

REFERENCE

*1.* J. M. Williams and C. L. Huffine, *Nuclear Sci. and Eng.* **9**, 500 (1961).

Frank Kerze, Jr.

*Division of Reactor Development*
*U. S. Atomic Energy Commission*
*Washington, D. C.*
*Received June 8, 1961*

## A Generalized Equivalence Theorem For Resonance Escape in Heterogeneous Systems

Chernick and Vernon (*1*) have outlined an equivalence relationship between resonance absorption in heterogeneous and homogeneous systems. (According to them the suggestion of an equivalence was first advanced by Bakshi.) The development in (*1*) is based on the use of Wigner's rational approximation for the escape probability. In the following note it is shown that a general equivalence relationship can be written directly in terms of the escape probability.

Consider a fuel lump (or fuel lumps) of material with total cross section per fuel atom $\sigma_t(E)$, and scattering cross sections per fuel atom $\sigma_s(E)$. Let the fuel lump be embedded in a moderator of constant (scattering) cross section $\sigma_{m1}$. Then the average flux in the fuel lump $\phi_0$ is given by the expression

$$\begin{aligned} V_0\,\phi_0(E)\sigma_t(E) &= V_0\,P_0'(E)\int_E^{E/\alpha_0}\frac{\sigma_s(E)}{1-\alpha_0}\,\phi_0(E)\,\frac{dE}{E} \\ &\quad + \frac{N_1}{N_0}\,V_1[1-P_1(E)]\int_E^{E/\alpha_1}\frac{\sigma_{m1}\,\phi_1(E)}{1-\alpha_1}\,\frac{dE}{E} \end{aligned} \tag{1}$$

The zero subscript refers to events in the fuel lump and the subscript 1 refers to the moderator. It is assumed that the flux $\phi_1$ in the moderator is uniform in space and that the scattered flux is isotropic. $P_0'(E)$ is the fraction of neutrons of energy $E$ born within the fuel lump which have their next collision in the lump. $[1 - P_1(E)]$ is the probability that a neutron born in the moderator does not suffer its next collision in the moderator. $N_1$ is the moderator atom density and $N_0$ the atom density of the fuel.

In the narrow resonance approximation, the resonance width is assumed to be negligible compared to the energy spread between $E$ and $E/\alpha_1$. In this case, letting $\phi_1(E) = K/E$ the last term of (1) becomes

$$\begin{aligned} \frac{N_1V_1}{N_0}\,(1-P_1)\int_E^{E/\alpha_1}\frac{\sigma_{m1}\,\phi_1}{1-\alpha_1}\,\frac{dE}{E} \\ = \frac{N_1}{N_0}\,\sigma_{m1}\,V_1(1-P_1)\,\frac{K}{E} \end{aligned} \tag{2}$$

Now, as pointed out by Chernick, it is easily shown from (2) that $N_1\sigma_{m1}V_1(1 - P_1) = N_0\sigma_t V_0(1 - P_0)$; where $(1 - P_0)$ is the escape probability for a *uniform* isotropic source distribution in the fuel lump. Thus, if the assumption is made that $P_0' = P_0$ (1) can be written

$$\frac{\phi_0(E)\sigma_t(E)}{P_0(E)} = S_0(E) + \frac{K}{E}\,\frac{\sigma_t(E)[1-P_0(E)]}{P_0(E)} \tag{3}$$

or

$$\phi_0\left(\sigma_t + \sigma_t\,\frac{(1-P_0)}{P_0}\right) = S_0 + \frac{K}{E}\,\sigma_t\left(\frac{1-P_0}{P_0}\right) \tag{3a}$$

where

$$S_0(E) = \int_E^{E/\alpha_0}\frac{\sigma_s(E)}{1-\alpha_0}\,\phi_0(E)\,\frac{dE}{E}$$

In the homogeneous case the corresponding equation is

$$\phi(\sigma_t + \sigma_m) = S_0 + \frac{K}{E}\,\sigma_m \tag{4}$$

where $\sigma_m$ is the moderator cross section per fuel atom. Comparison of (3a) and (4) shows that the expressions are formally identical if $\sigma_T[(1 - P_0)/P_0]$ is identified with $\sigma_m$. Thus, the generalized equivalence relationship states that