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Optimum Control and Flat Flux

It was shown by Dr. G. Goertzel in his important investigations of the minimum critical mass problem (*1*) that in a homogeneous thermal reactor with constant moderating and thermal transport properties throughout core and reflector the minimum critical mass is achieved if the fuel is distributed so that the thermal flux in the core is flat. Subsequently it was pointed out (*2*) that a formally quite similar problem exists, that of optimum control: the amount of poison introduced into a homogeneous thermal reactor is minimal if the control rods are arranged in a central control region in such a way that the thermal flux is flat; again it is assumed that the moderating and thermal transport properties are constant throughout the controlled and the uncontrolled regions.

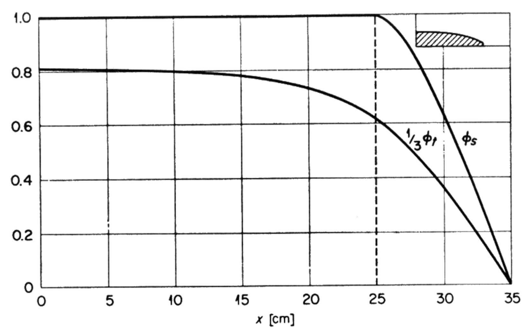

FIG. 1. Slow and fast fluxes in a MTR type slab reactor with optimal control. The poison distribution is shown in the inset.

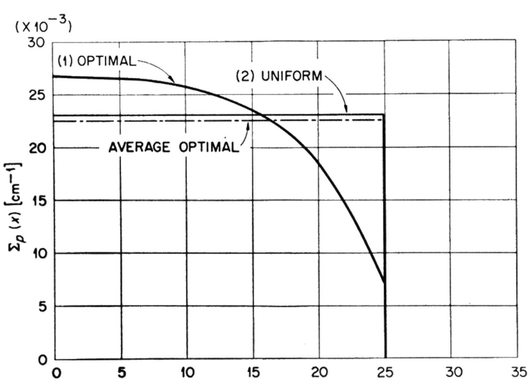

FIG. 2. Poison distributions required for criticality within the optimal size control region of a slab reactor (two-group).

Indeed, both cases—optimum control as well as minimum critical mass—are covered by a theorem which was proved by Dr. Goertzel (*1*), using the methods of the calculus of variations: if H is a symmetric operator, $\rho(\mathbf{x})$ a nonnegative function normalized so that $\int d\mathbf{x}\rho(\mathbf{x}) = 1$, and $\varphi_0(\mathbf{x})$ the fundamental mode of the eigenvalue equation

$$\varphi_0 = \lambda_0 H(\rho\varphi_0), \tag{1}$$

then λ_0 will be smallest if ρ is a constant. In the optimal control problem, the relations between the thermal flux φ and the slowing down density q(thermal) are

$$q = \mathfrak{L}_f \eta \Sigma_c \varphi$$

and

$$\varphi = \mathfrak{L}_s(q - \Sigma_c\varphi - \Sigma_p\varphi) \tag{2}$$

where $\mathfrak{L}_s$ is the diffusion operator, describing the effect of the introduction of a thermal neutron at one point in the reactor on the thermal flux at all the other points; $\mathfrak{L}_f$ is a similarly defined slowing down operator; Σ_c is the capture cross section of fuel and moderator; and Σ_p that of the control rods. A normalized poison density is defined by

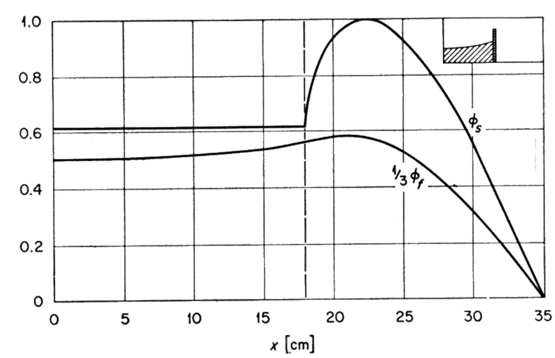

FIG. 3. Fluxes in a slab reactor with flat thermal flux in the undersized control region. The areas under the curves in the inset are representative of the ratio of continuous to lumped poison.

$$\Sigma_p/\Sigma_c = \langle \Sigma_p/\Sigma_c \rangle_{\text{avg}}\ \rho(\mathbf{x}) = -\lambda\rho(\mathbf{x}),$$

where the average is taken over the control region. This, together with Eq. (2), gives us the reactor equation for the thermal flux:

$$\varphi = -\langle \Sigma_p/\Sigma_c \rangle_{\text{avg}}[1 - \Sigma_c(\mathfrak{L}_s\mathfrak{L}_f\eta - \mathfrak{L}_s)]^{-1}\mathfrak{L}_s(\rho\varphi) = \lambda H(\rho\varphi). \tag{3}$$

Thus, the theorem which we have quoted above applies also to this case, always provided that the operator H is symmetric. An example in point is the multigroup formalism, in which H is symmetric since it is composed of Green's functions of self-adjoint differential equations.

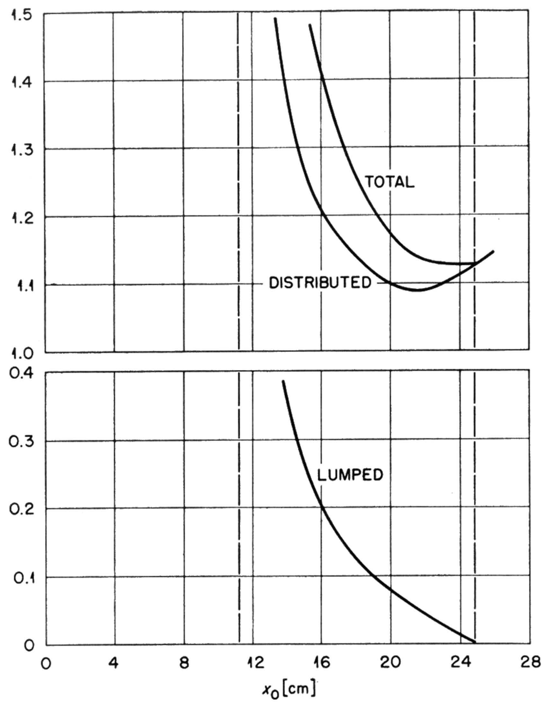

FIG. 4. Dependence of the poison investment $\int dx\Sigma_p(x)$ required for the maintenance of criticality in a slab reactor of width 2 × 35 [cm] on the size of the control region 2 x_0.

As an example, we have performed a two-group calculation for a slab reactor of MTR composition. Once the over-all size of the reactor has been decided upon, the size of the control region and the optimum poison distribution inside it are fixed. The fast and slow fluxes and currents are continuous everywhere inside the reactor (Fig. 1). The poison distribution is shown in the inset, and also in Fig. 2, curve (1). The mean value of curve (1) is indicated by the dot-and-dash line. A uniform poison distribution which would maintain criticality would have to have the strength given by curve (2).

If in a reactor of fixed size we decrease the extent of the control region, keeping the thermal flux inside it flat, the reactor will go supercritical if we do not add lumped poison at the boundary of the control region. The poison in our example absorbs only thermal neutrons. Therefore, the first derivative of the thermal flux has a discontinuity at the control region boundary (Fig. 3). The total poison investment is indeed greater than in the optimal case, as we can see from Fig. 4. Similarly, if the size of the control region is increased beyond the optimal dimensions, the reactor will only remain critical, and the flux flat, if lumped sources are attached at the control region boundaries. However, it turns out that the optimum found by means of the calculus of variations is only a stationary point; if we interpret sources as negative poison, lumping at the boundaries leads to less total "poison" than the optimal distribution does.

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