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Average Flux In Reactor Test Volumes

The prediction of reaction rates of neutrons with nuclei within a finite sample in a research or test reactor requires the computation of the average neutron flux over the region. Formulas are presented in this note that facilitate such evaluations for off-center samples.

Cylindrical symmetry of the basic reactor flux distribution is assumed. The functions that make up $\phi(r)$ are known, for example, by previous one- or two-group calculations.

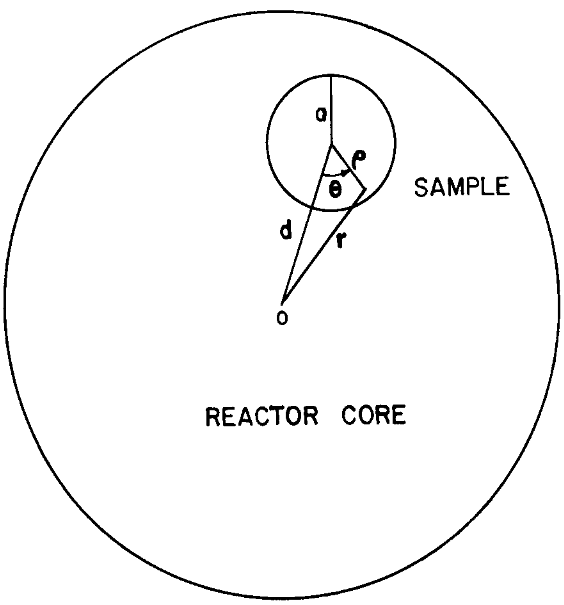

FIG. 1

The flux distribution in the sample of radius a is given either by $\phi(r)$ or $\phi(\rho, \theta)$. Coordinates are related as in Fig. 1, where

$$r^2 = \rho^2 - 2\rho d \cos\theta + d^2.$$

A Taylor's series expansion in ρ is made about an origin at the center of the sample.

$$\phi(\rho, \theta) = \sum_{n=0}^{\infty} \phi^{(n)}(0, \theta)\rho^n/n!$$

The first derivative is

$$\phi'(\rho, \theta) = [\partial\phi(r)/\partial r](\partial r/\partial\rho)$$

where $\partial r/\partial\rho = (\rho - d\cos\theta)/r$. Thus

$$\phi'(0, \theta) = -\phi'(d)\cos\theta.$$

Successive differentiation yields higher order terms. Integration of $\phi(\rho, \theta)$ over ρ from 0 to a and of θ from 0 to 2π is performed to determine the average flux over the area. Use is made of the fact that all coefficients of odd powers of r in the series vanish. The final expression for the average flux in terms of the known flux distribution is

$$\bar{\phi} = (\pi a^2)^{-1}\int \phi(\rho,\theta) r dr d\theta = \sum_{n=0}^{\infty} \frac{A_{2n}(a/2)^{2n}}{n!\,(n+1)!}.$$

The first coefficients of the series, sufficient for most purposes, are found to be

$$A_0 = \phi$$

$$A_2 = \frac{\phi'}{d} + \phi''$$

$$A_4 = \frac{\phi'}{d^3} - \frac{\phi''}{d^2} + \frac{2\phi'''}{d} + \phi''''$$

$$A_6 = \frac{9\phi'}{d^5} - \frac{9\phi''}{d^4} + \frac{6\phi'''}{d^3} - \frac{3\phi''''}{d^2} + \frac{3\phi'''''}{d} + \phi''''''$$

where all derivatives of $\phi(r)$ are evaluated at d.

When, as in two-group theory, the flux in the core or reflector is represented by a combination of Bessel functions, $J_0(r)$, $Y_0(r)$, $I_0(r)$, $K_0(r)$, integrals based on Gegenbauer's addition formulas (*1*) are more convenient than the general series. For the geometry of the figure

$$J_0(r) = J_0(\rho)J_0(d) + 2\sum_{p=1}^{\infty} J_p(\rho)J_p(d)\cos p\theta$$

$$Y_0(r) = J_0(\rho)Y_0(d) + 2\sum_{p=1}^{\infty} J_p(\rho)Y_p(d)\cos p\theta$$

$$I_0(r) = I_0(\rho)I_0(d) + 2\sum_{p=1}^{\infty} (-1)^p I_p(\rho)I_p(d)\cos p\theta$$

$$K_0(r) = I_0(\rho)K_0(d) + 2\sum_{p=1}^{\infty} I_p(\rho)K_p(d)\cos p\theta$$

Integration over θ causes all terms in the infinite series portion of the functions to vanish by virtue of the orthogonality of the cosines. Thus, integration over ρ for the first term only is required. It is noted that for purposes of averaging, the flux in the sample drops off as $J_0(\rho)$ or increases as $I_0(\rho)$ from its value at the center of the sample.

TABLE I

Flux function	Average over sample
$J_0(\mu r)$	$J_0(\mu d)$ · $\frac{2J_1(\mu a)}{\mu a}$
$Y_0(\mu r)$	$Y_0(\mu d)$ · $\frac{2J_1(\mu a)}{\mu a}$
$I_0(\kappa r)$	$I_0(\kappa d)$ · $\frac{2I_1(\kappa a)}{\kappa a}$
$K_0(\kappa r)$	$K_0(\kappa d)$ · $\frac{2I_1(\kappa a)}{\kappa a}$

This suggests that ratios of average to central fluxes as in reflected reactor cores may be utilized (*2*). For example, in the case of the $J_0(\mu r)$ that appears in the two-group fast or thermal core flux, this ratio is $\bar{\phi}/\phi_c = 2J_1(x)/x$ where $x = \mu a$.

Table I gives all the averages of interest in a reflected core with or without a central control rod. It is evident that the second relation may also be applied in the core to the functions $I_0(\nu r)$, and K_0 (νr), with κ replaced by ν in the modified Bessel functions. It may be shown by the use of various recursion formulas for Bessel functions that these results are consistent with those obtained by the general series.

The average thermal flux is computed as a numerical illustration of the use of the formulas for a 6-cm radius circular region with center at a distance of 15 cm from the axis of a one-group reflected core. Let the relative flux be $\phi_2(r) \sim J_0(0.08r)$. The flux at the sample center is $J_0(1.2) = 0.6711$; this is multiplied by $2J_1(0.48)/0.48 = 0.9715$, to give an average of 0.6520.

This problem arose in the evaluation of irradiations in the Westinghouse Testing Reactor, at the suggestion of Mr. M. A. Schultz.

REFERENCES

(*1*) A. Gray, G. B. Mathews, and T. M. MacRobert, "Bessel Functions," p. 71. Macmillan, New York, 1931.
(*2*) R. L. Murray, "Nuclear Reactor Physics," p. 80. Prentice-Hall, New York, 1957.

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