Letters to the Editor

Comments on "Laser Fusion Hydrodynamics Calculations"

Moses[1] recently discussed the applicability of the multigroup flux-limited diffusion theory used in LASNEX for the transport of nonthermal charged particles. We conducted a similar analysis for point monoenergetic ion sources embedded in infinite plasmas.[2] For these simple geometries, the moments method provides "exact solutions" of the Fokker-Planck equations, including the ion deflection term. In such benchmark problems, the usual difficulties due to anisotropic fluxes, sharp ion distributions in space, etc., remain, and the diffusion theory can be in error. For the transport of alpha particles in deuterium-tritium (DT) plasmas, we fully agree with Moses. The straight line trajectories will be a better approximation than the diffusion formalism. As a matter of fact, by using the Green's functions provided by the moments method, we could reproduce his Monte Carlo results. (See Fig. 11 of Ref. 1 and Figs. 9 and 10 of Ref. 2.)

Unfortunately, for lighter particles (protons) and/or heavier plasmas, the ion scattering becomes more and more important. On the basis of the Fokker-Planck equation, we have shown that the energy deposition for point monoenergetic sources can be computed from the universal Green's function. One can write

$$4\pi s^2 I_{pt}(s)\cdot\frac{s_M}{E_s} = G_{pt}(\xi;\alpha,\beta) \quad , \tag{1}$$

where

$I_{pt}(s)$ = energy deposited on plasma ions or electrons ($\mathrm{MeV\cdot cm^{-3}\cdot s^{-1}}$)

s = distance from the point source (cm)

s_M = maximum range of fast ions (cm)

E_s = energy of the ion source (MeV).

The Green's function expressed in terms of reduced distances

$$\left(\xi = \frac{s}{s_M} \leq 1\right)$$

is defined by *only two parameters*:

$$\alpha = \frac{m_i^*}{4m}$$

and

$$\beta = \frac{4}{3}\left(\frac{m_e}{\pi m}\right)^{1/2}\cdot\frac{m_i^*}{m z_i^*}\,\frac{\log\Lambda_e}{\log\Lambda_i}\left(\frac{E_s}{\theta_e}\right)^{3/2} \quad , \tag{2}$$

where

m, m_i^*, m_e = mass of fast ions, plasma ions, and electrons, respectively

θ_e = electron temperature (MeV)

$\log\Lambda_e$, $\log\Lambda_i$ = usual Coulomb logarithms.

The scattering parameter, α, is assumed to be zero in the Bragg approximation proposed by Moses, and β represents the electron contribution to the slowing down process. Two extreme cases are shown in Figs. 1 and 2.

The transport of 3.5-MeV alpha particles in a 50-keV DT plasma exhibits little influence by ion scattering (α = 0.15), since the exact solution and the Bragg profile are very close (Fig. 1). On the contrary, for the penetration of 1-MeV protons in boron hydride plasmas ($BD_{1.5}T_{1.5}$), the ion scattering (α = 2) plays an important role (Fig. 2). Many results that are not given in Ref. 2 lead to the conclusion that the straight line trajectory approximation is no more valid when $\alpha > 0.5$. On the other hand, we have recently made comparisons with diffusion theory. For a large number of groups, this theory is equivalent to age theory. The corresponding Green's function takes a very simple form when $\beta = 0$:

$$G_{\mathrm{age}}(\xi;\alpha,\beta=0) = \frac{3\alpha}{\sqrt{\pi}}\int_{(3\alpha/2)^{1/2}\cdot\xi}^{\infty}\left(1-\frac{3\alpha}{2}\,\frac{\xi^2}{t^2}\right)^{-3/4} e^{-t^2}dt \quad . \tag{3}$$

A few results are plotted together with the exact solution in Fig. 3. For small ξ values, the age theory curves have the uncorrect behavior as expected, since the diffusion theory is known to be inaccurate near the sources. For deep penetrations (large ξ), the distributions exhibit unphysical tails because the diffusion approximation smears out the correlation between space and energy, which is a typical

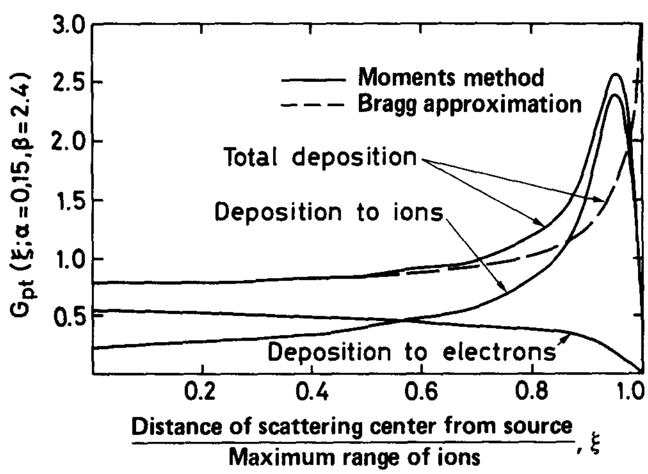

Fig. 1. Deposition of 3.5-MeV alpha energy from a point source to D-T plasma.

[1] G. A. MOSES, *Nucl. Sci. Eng.*, **64**, 49 (1977).

[2] P. A. HALDY and J. LIGOU, *Nucl. Fusion*, **17**, 6 (1977).

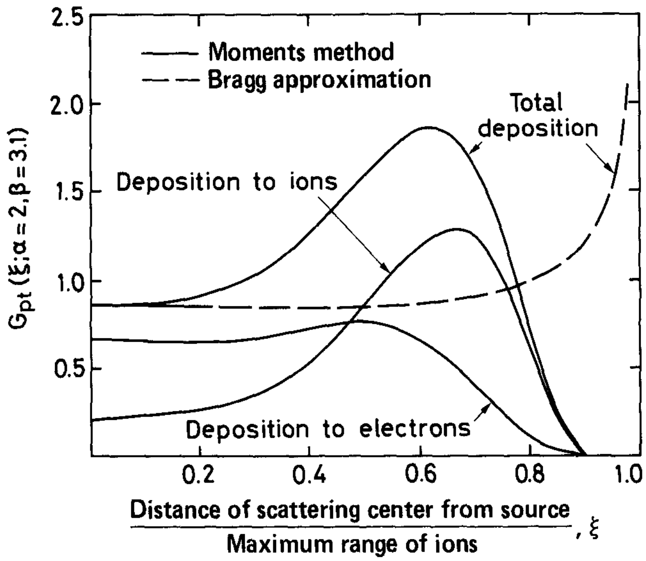

Fig. 2. Deposition of 1-MeV proton energy from a point source to BDT plasma.

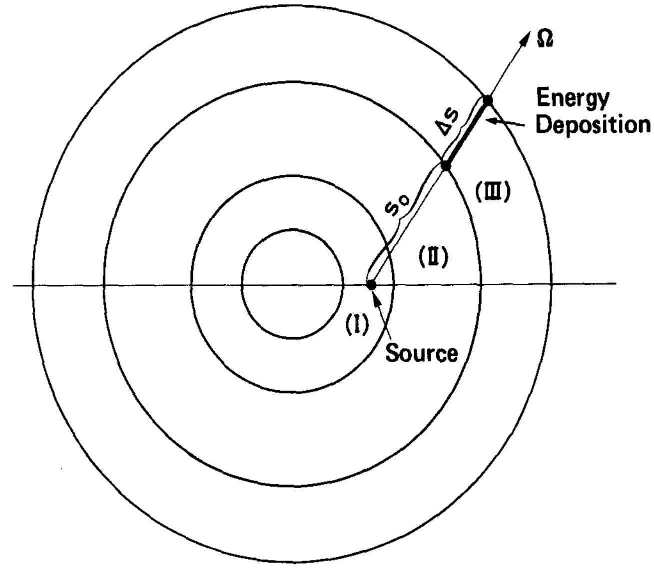

Fig. 4. Energy deposition in a multishell system.

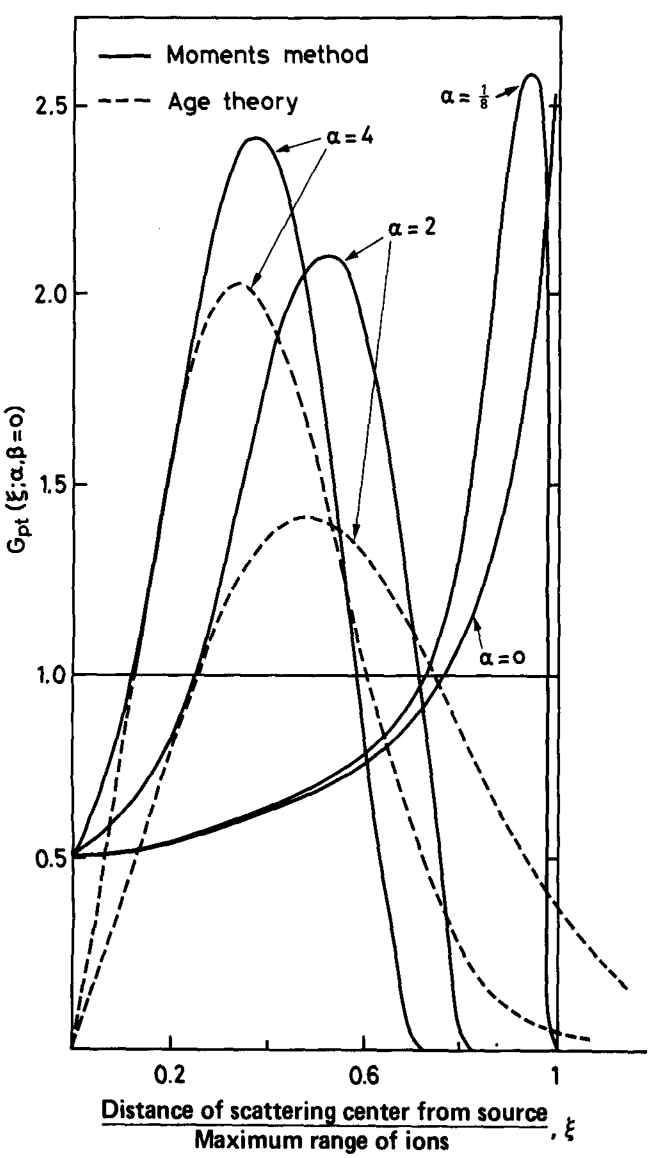

Fig. 3. Deposition of fast ion energy to plasma for a point monoenergetic source ($\beta = 0$).

feature of the continuous slowing down model. Nevertheless, for $\alpha \geq 4$, the diffusion theory gives reasonable results, and the flux-limited correction should reduce the discrepancies.

A general conclusion can be drawn: For α values between 0.5 and 3, neither the straight line approximation nor the flux-limited diffusion is suitable. The best we can do to avoid Monte Carlo calculations is to include in hydrodynamics codes the Green's function $G_{pt}(\xi;\alpha,\beta)$ in a tabulated or fitted form.

In a one-dimensional code, the spherical plasma is divided into concentric zones. Assuming as Moses did that fusion reaction products originate at the center of Zone I, one can calculate, for example, the energy deposition W_z in Zone III (Fig. 4):

$$W_z = \iint_{\text{Zone III}} I_{pt}(s)s^2 ds d\Omega \quad ,$$

or, by using Eq. (1),

$$\begin{cases} W_z = \int_{4\pi} \frac{d\Omega}{4\pi} W_z(\mathbf{\Omega}) \\ W_z(\mathbf{\Omega}) = E_s \int_{\xi_0}^{\xi_0+\Delta\xi} G_{pt}(\xi;\alpha,\beta)\, d\xi \end{cases}$$

$$\cong E_s \Delta\xi \left[G_{pt}(\xi_0) + \frac{1}{2} G'_{pt}(\xi_0)\Delta\xi \right] \quad ,$$

where

$$\xi_0 = \frac{s_0}{s_M} \text{ and } \Delta\xi = \frac{\Delta s}{s_M} \quad .$$

Then, $W_z(\mathbf{\Omega})$ is the energy deposition in Zone III for a point source (E_s) in Zone I and in the direction $\mathbf{\Omega}$. It has the same meaning as ΔE given by Moses in his Eq. (42) provided that the Green's function previously determined by this method, which includes ion scattering, is not more complicated than the approaches based on straight line trajectories.

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