

Letters to the Editor

Comments on Particle Transport in Finite Rods

In an interesting exchange, Woolf et al.¹ have derived expressions by the method of invariant imbedding for the contributions to the reflected and transmitted particle currents for a finite one-dimensional rod, and Williams² has pointed out how these relations can be obtained by a generating function technique. Woolf et al. define $T_n(t)$ and $R_n(t)$ as the transmitted and reflected currents, respectively, of n -times scattered particles for a rod of length t . The generating functions of Williams are

$$G(z;t) = \sum_{n=0}^{\infty} z^n T_n(t) \quad , \quad (1)$$

$$H(z;t) = \sum_{n=0}^{\infty} z^n R_n(t) \quad . \quad (2)$$

We wish to point out that these expansions are not mere mathematical artifacts, but have an immediate physical interpretation. If the parameters b and f of Woolf et al. are normalized to be the *relative* probabilities of backward and forward scatterings (so that $f + b = 1$) and if z is taken as the scattering probability, then G and H are, respectively, the total transmitted and reflected currents. The currents themselves serve as generating functions. An order-of-scattering expansion is nothing but an expansion in the scattering probability. Williams' Eqs. (3) and (4) are the well-known Riccati equations applied to this case.

One can readily obtain still other relations. For instance, the internal currents in the positive and negative directions at any point x , $0 \leq x \leq t$, are, respectively,

$$\phi_+(x; z) = \frac{(A^2 - B^2)^{1/2} \cosh(A^2 - B^2)^{1/2} (t - x) - A \sinh(A^2 - B^2)^{1/2} (t - x)}{(A^2 - B^2)^{1/2} \cosh(A^2 - B^2)^{1/2} t - A \sinh(A^2 - B^2)^{1/2} t} \quad , \quad (3)$$

$$\phi_-(x; z) = \frac{B \sinh(A^2 - B^2)^{1/2} (t - x)}{(A^2 - B^2)^{1/2} \cosh(A^2 - B^2)^{1/2} t - A \sinh(A^2 - B^2)^{1/2} t} \quad . \quad (4)$$

We obtained these results by a trivial substitution in the general solution³ obtained by the Transfer Matrix Method, but they can be readily derived by many other methods as well.

Note that with our interpretation, z is restricted for a nonmultiplying system, so that $A < 0$.

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¹STANLEY WOOLF, JOHN C. GARTH, and WILLIAM L. FILIPPONE, *Nucl. Sci. Eng.*, **62**, 278 (1977).

²M. M. R. WILLIAMS, *Nucl. Sci. Eng.*, **63**, 357 (1977).

³RAPHAEL ARONSON, *J. Math. Phys.*, **11**, 931 (1970).

Comments on Beryllium ($n, 2n$) Cross Sections in ENDF/B-IV and -V

In a recent paper by Drake et al.,¹ a comparison was made between their experimental results and the evaluated beryllium cross sections in ENDF/B-IV (Ref. 2). As the authors indicated, the agreement was good for the integral elastic and ($n, 2n$) cross sections at their three incident energies of 5.9, 10.1, and 14.2 MeV; however, the double-differential cross sections, $\sigma(E \rightarrow E', \mu)$, were in disagreement. Drake correctly pointed out that the low-lying ⁹Be levels were overemphasized in the evaluation. This problem is one that can be and has been corrected in subsequent evaluations. However, there are inherent difficulties with the ENDF/B formats and Cross Section Evaluation Working Group (CSEWG) procedures that carry over to ENDF/B-V (Ref. 3) and that limit the usefulness of the beryllium evaluation. This Letter points out these problem areas.

It is well known^{4,5} that levels in ⁹Be decay by neutron emission to levels in ⁸Be, so that the ($n, 2n$) reaction can be described as a time-sequential process, ⁹Be(n, n_1) × ^{9*}Be(W_9) (n_2) ^{8*}Be(W_8), where the W 's are the excitation energies corresponding to levels in ⁹Be and ⁸Be that are, for the most part, very wide.⁵ In 1973, the ENDF/B formats were changed to allow the ⁹Be($n, 2n$) reaction to be described by up to four such time-sequential processes. All ⁹Be levels were to be considered as having zero width, and the energy-angle correlation was neglected for the second neutron. The validity of neglecting the energy-angle correlation has been discussed previously.⁶ The data were presented in this form for ENDF/B-IV using four levels in ⁹Be. The second neutron energy and angle distributions, presented as tables, included the wide-level effects of both nuclei, since they had been previously integrated over the level distribution functions in both ^{9*}Be and ^{8*}Be. There is another option within ENDF/B formats that would permit the correlated distribution, $\sigma(E \rightarrow E', \mu)$, to be entered directly, but the CSEWG procedures do not allow this format to be used.

In 1976, both Drake's work¹ and the results of another recent measurement⁷ were made available to us, and ⁹Be was reevaluated. These results were placed in the ENDL

¹D. M. DRAKE, G. F. AUCHAMPAUGH, E. D. ARTHUR, C. E. RAGAN, and P. G. YOUNG, *Nucl. Sci. Eng.*, **63**, 401 (1977).

²R. J. HOWERTON and S. T. PERKINS, "Evaluated Neutron-Interaction and Gamma-Ray Production Cross Sections of ⁹Be for ENDF/B-IV Mat. No. 1289," UCRL-51603, Lawrence Livermore Laboratory (1974).

³R. J. HOWERTON and S. T. PERKINS, unpublished beryllium evaluation (1976).

⁴S. T. PERKINS, "The Be($n, 2n$) Reaction and Its Influence on the Age and Fast Effect in Beryllium and Beryllium Oxide," AN-1443, Aerojet General Nucleonics (1965).

⁵C. L. COCKE and P. R. CHRISTENSEN, *Nucl. Phys.*, **A111**, 623 (1968).

⁶S. T. PERKINS, *Nucl. Sci. Eng.*, **31**, 156 (1968).

⁷F. O. PURSER, Triangle Universities Nuclear Laboratory, Private Communication (1976).

file,⁸ which has no restriction on the number of levels involved in the time-sequential process or the width of the levels. The new evaluation used excitation of five levels in ⁹Be, four of which were wide. Comparison of these results with Drake's results (see Fig. 6 of Ref. 1) showed good agreement except in the deep minima. Likewise, a comparison of calculated and experimental 14-MeV ⁹Be pulsed-sphere⁹ neutron emission spectra showed agreement within experimental uncertainty.

The difficulties in the ⁹Be(*n*, 2*n*) reaction were presented to the CSEWG Codes and Format Committee in the fall of 1976, and a request made to change the ENDF/B format to allow the reaction to be described by up to five levels, wide if necessary, in ⁹Be. This request was disapproved by the Committee, and as a result, the ⁹Be(*n*, 2*n*) reaction in ENDF/B-IV and -V are identical.

⁸R. J. HOWERTON et al., "The LLL Evaluated Nuclear Data Library (ENDL)," UCRL-50400, Vol. 15, Pt. A-D, Lawrence Livermore Laboratory (1975-1977).

⁹C. WONG et al., "Livermore Pulsed Sphere Program: Program Summary Through July 1971," UCRL-51144, Rev. 1, Lawrence Livermore Laboratory (1972).

There is, however, a warning to the potential user in the ENDF/B-V descriptive data: "Any application of this file with incident neutron energy above 5 MeV can yield questionable results."

These two recent experiments^{1,7} on ⁹Be, specifically measuring $\sigma(E \rightarrow E', \mu)$, have proven to be very useful to the evaluator in determining the breakup modes for the ⁹Be(*n*, 2*n*) reaction. Since this material will be a major constituent in controlled thermonuclear reactor applications, additional similar experiments would be even more useful. However, until the ENDF/B format and CSEWG procedures are changed to allow the breakup to be properly described, the evaluation we did cannot be represented in the ENDF/B-V files. *Caveat Emptor*.

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