Letters to the Editor

Transport Eigenvalues

The Note by Ronen et al.[1] discussing different eigenvalue forms of the Boltzmann neutron transport equation was welcome. It contains, however, an implication that I think bears further consideration, that $1/v$ absorption hardens the energy spectrum. The authors quote Bell and Glasstone[2]; a similar remark has often been made (e.g., Ref. 3).

Surely a more precise exposition is as follows. Absorption removes neutrons. If the absorption cross section is '$1/v$,' this removal is at a local rate

$$(\Sigma_0^a/v)\,\phi(\boldsymbol{r},E) = (\Sigma_0^a/v)vn(\boldsymbol{r},E) = \Sigma_0^a n(\boldsymbol{r},E) \quad ,$$

where Σ_0^a is a constant. Then the rate of removal is everywhere proportional to the existing neutron density in position and energy. As a consequence, the neutron density and flux both tend to decrease in time, but with no change in spectrum or, indeed, in shape.

If, of course, one is bold enough to *replace* these lost neutrons, to maintain a steady state perhaps, the spectrum may well change because of the energy distribution of the source neutrons so introduced. In many cases, these are high-energy neutrons, compared with the partially thermalized spectrum that we know the Boltzmann operator would lead to in the absence of losses, so the spectrum is hardened. But if $1/v$ absorption were compensated by neutrons from a cold thermal column, for example, the spectrum would be softened.

I suggest, therefore, that we avoid saying that $1/v$ absorption hardens the spectrum; it is too compressed and could be fallacious. Naturally, the situation with non-$1/v$ absorption is different again and, dependent on the departure of the cross section from this ideal, might of itself soften or harden the spectrum.

In their calculations (which are for a practical rather than a theoretical case of spectrum-independent 'B' operator in Ref. 1), Ronen et al. do find a hardening in the α-eigenvalue formulation. It would be interesting to learn whether this is the immediate consequence of the additional source neutrons or of the leakage term, which is clearly not $1/v$.

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May 18, 1976

[1]Y. RONEN, D. SHVARTS, and J. J. WAGSCHAL, *Nucl Sci Eng.*, **60**, 97 (1976).

[2]G. I. BELL and S. GLASSTONE, *Nuclear Reactor Theory*, Van Nostrand Reinhold Company, New York (1970).

[3]D. J. BENNETT, *Elements of Nuclear Power*, p. 49, Longmans, London (1972).

The Reactivity Dependence of Neutron Energy Spectra

The communication by Lewins,[1] which was prompted by a Note[2] on different eigenvalue formulations of the transport equation, gives us an appropriate opportunity to clarify and elaborate on the observation in that Note, that the higher the α-reactivity, the harder the spectrum. More precisely, the meaning of this statement is that, considering a series of homogeneous multiplying assemblies of a given composition, their α-spectra, i.e., the energy spectra obtained by solving the equation

$$\boldsymbol{\Omega}\cdot\nabla\psi + (\Sigma + \alpha/v)\psi = \int c(E')\Sigma(E')f(\boldsymbol{v}'\to\boldsymbol{v})\psi(\boldsymbol{v}')d^3v' \quad , \tag{1}$$

become harder with (algebraically) increasing α. The number of secondary neutrons per collision is $c(E')$, and their normalized distribution is $f(\boldsymbol{v}'\to\boldsymbol{v})$. Since the term α/v in Eq. (1) is equivalent to varying the concentration of a $1/v$ absorber, this observation is sometimes misinterpreted or misphrased.[2,3] This effect was demonstrated in the Note[2] for unreflected ^{239}Pu spheres of constant density and increasing radii. But it is just as true for any geometry and for other changes affecting the reactivity, for instance, varying the density or varying both density and size. The increase in reactivity, which is accompanied by a hardening of the neutron spectrum, is compensated in Eq. (1) by adding extra fictitious $1/v$ adsorption. As was correctly pointed out by Lewins,[1] the change in $1/v$ absorption alone does not modify the neutron spectrum.

As a matter of fact, we would first like to discuss Lewins' argument about pure $1/v$ absorption and present it in a more formal way. Let us then express the full (nonstationary) linear transport equation as

$$1/v\,\dot{\psi} + O\psi = 0 \quad , \tag{2}$$

where the operator O represents all time-independent terms. If we now homogeneously add an α/v absorber to the given assembly, the modified flux, ψ^*, will satisfy the equation

$$1/v\,\dot{\psi}^* + (O + \alpha/v)\psi^* = 0 \quad . \tag{3}$$

The immediate solution of this equation, in terms of ψ, is

$$\psi^* = \psi\exp(-\alpha t) \quad , \tag{4}$$

which explicitly demonstrates that the neutron energy spectrum is not affected by adding the $1/v$ absorber to the original assembly. We further observe that the α spectrum of a noncritical assembly is the true (physical) unmodified asymptotic spectrum of the assembly.

We now return to the main point of this Letter, the point

[1]J. LEWINS, *Nucl Sci Eng.*, **62**, 180 (1977).

[2]Y. RONEN, D. SHVARTS, and J. J. WAGSCHAL, *Nucl. Sci. Eng.*, **60**, 97 (1976).

[3]G. I. BELL and S. GLASSTONE, *Nuclear Reactor Theory*, p. 47, Van Nostrand Reinhold Company, New York (1970).