before), one has

$$\alpha_1\beta_2 \ll \alpha_2\beta_1 \quad , \qquad (20)$$

and

$$\left(\frac{\alpha_1+\beta_2}{2}\right)^2 \ll \alpha_2\beta_1 \quad . \qquad (21)$$

It therefore follows that

$$\lambda \gtrsim (\alpha_2\beta_1)^{1/2} \quad . \qquad (22)$$

From Eqs. (4) and (5) one can see that this approximation is equivalent to neglecting the fission neutron source term in Eq. (4), it being small in comparison to the fusion neutron source term, and neglecting the fusion reaction heat source term in Eq. (5) in comparison to the fission reaction heat source term.

With the definition of f [Eq. (15)] one has from Eq. (22)

$$f(x) \gtrsim 4.26\left[\frac{x^2(1-x)}{1+3.3x}\right]^{1/2} T_0^{1.68} \quad . \qquad (23)$$

This function has a maximum for $x = 0.57$ where

$$f \gtrsim 0.94\, T_0^{1.68} \quad . \qquad (24)$$

Assume, for example, that T_0 = 5 keV; then f = 14.2 and ν^* = 28.0. For T_0 = 10 keV, f = 45.0 and ν^* = 86.5.

With this computed value for f, one can estimate the reduction in the required critical mass due to the bootstrap coupling of the fission chain reaction with the fusion process.

Since the critical radius is proportional to $(\nu - 1)^{-1/2}$, it is reduced by the factor $f^{1/2}$ and the critical mass reduced by the factor $f^{3/2}$. For f = 45 the critical mass is reduced by ≃300. To attain a large f-value, the pellet must be heated from T = 0 to $T \approx 10$ keV, which requires an energy approximately 10 times the compression energy, such that an overall reduction in the energy of ~30 is required to achieve criticality.

The effect of the fission-fusion bootstrap on the exponential growth of the chain reaction is even more pronounced. The exponential growth of the chain reaction is determined by the Rossi-α, which is proportional to $(\nu - 1)$. In the case of the fission-fusion chain reaction, Rossi-α has to be multiplied by the factor f (=45). Since the relative yield is determined by the factor $R\alpha$, where R is the pellet radius, an increase in α by a factor 45 would permit an equal reduction in R to achieve the same relative yield. This would imply a reduction in the pellet volume by a factor $(45)^3$ ($\sim 10^5$). Reference 1 gives a value for the compression energy of $\sim 2 \times 10^8$ J required to achieve a substantial yield. This compression energy could thus be reduced by $\sim 10^5$. Simultaneous heating from T = 0 to T = 10 keV requires ~10 times more energy so that an overall reduction in energy by $\sim 10^4$ (that is, from 2×10^8 to 2×10^4 J) can be achieved.

In addition to the coupling effect given here, there is an even more direct coupling as the fast-moving fission products kick off fusionable nuclei to attain kinetic energies required to overcome the fusion barrier. This effect becomes increasingly more important with higher pellet densities.

OTHER ASPECTS OF IMPORTANCE

Although the fission-fusion hybrid pellets can require as much trigger energy as pure fusion pellets, the character of the rapidly growing fission-fusion chain reaction leads to high burnup yield for both nuclear components. Since magnetohydrodynamic systems can convert the energy of the fireball into useful energy with a much higher efficiency then in conventional fission reactors, a much better utilization of the fissionable fuel is achieved. Furthermore, should the limited efficiency of laser trigger systems or the pulse shaping problems of relativistic electron beams pose a serious problem, as would be the case for laser- or e-beam fusion, the use of bunched-ion beams may become a very interesting alternative.[4]

F. Winterberg

Desert Research Institute
University of Nevada System
Reno, Nevada 89507

June 25, 1975

[4] F. WINTERBERG, *Plasma Phys.*, **17**, 69 (1975).

Reply to "Comments on 'Analysis of the Microfission Reactor Concept' "

THE COMPRESSION ENERGY

The relation $E = (2/3)pV$, quoted without explanation, gives the impression of being an exact result. (Note that it is "=" rather than "≃" in a majority of Winterberg's papers.) The only potentially applicable exact model is the degenerate electron gas for which $E = (3/2)pV$, and we assumed (as did all other workers with whom we discussed it) that this was the intended model. Of course, the difference is really not very significant. Our Thomas-Fermi-corrected calculation (the best simple model available) is theoretical over-kill, and the difference of 34% in energy for plutonium (or even the factor-of-2 differences for reflector materials) is small compared to other uncertainties in the calculation—or to the several orders of magnitude by which the whole scheme misses practicality.

REFLECTED PELLETS

The discussion here misses the essential point. The fact that Winterberg's estimated time for a reflected neutron to return to the fissionable core, $\sim 2 \times 10^{-10}$ sec, is shorter than the inertial confinement time is irrelevant. It is much *greater* than the e-folding time of the neutron population as a whole, given by the inverse of Rossi-α, which must be $\sim 3 \times 10^{-11}$ sec for explosive yields from pellets of the size considered here. Reflected neutrons simply arrive too late to have much effect on the diverging chain.

FISSION-FUSION PELLETS

While interesting, Winterberg's comments have no relation to our paper, which considered pure fission only. Without making detailed calculations (a far-from-trivial undertaking), it is impossible for us to do more than suggest that the argument presented is dangerously simplified, and the conclusion counter to our intuition.

R. K. Cole, Jr.
J. H. Renken

Theoretical Division 5223
Sandia Laboratories
Albuquerque, New Mexico 87115

July 14, 1975