retained and allowed to cancel. On the other hand, one may wish to know how much difference there is in some integral parameter due to two different ways of doing the reactor calculation. For example, one may wish to calculate the sodium worth in a critical facility with two different cross-section sets, neither of which predicts the critical mass correctly. The straightforward procedure would be to make a separate flux calculation with each cross-section set (obtaining different eigenvalues), calculate the sodium worth in each case, and subtract the two. The variational procedure, with the δk correction, would be appropriate in this case. Thus, the appropriate formalism in any particular case depends on just how the question is put, and the variational formalism seems to have sufficient generality to accommodate a variety of questions.

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Variational Versus Generalized Perturbation Theories—Are They Different?

The number and scope of applications of perturbation-theory formulations for integral parameters of the form of ratios of linear and bilinear functionals has greatly increased in recent years. There are several versions of these formulations that differ in the form of the perturbation expressions, in the approaches used for deriving these expressions, and in the terminology used to refer to them.

Usachev[1] and Gandini[2] have derived a generalized perturbation theory (GPT) on the basis of physical considerations. Their GPT formulations are restricted to alterations that leave the reactor critical. Using variational methods, Stacey[3,4] derived similar expressions that allow for alterations that change the static eigenvalue of the reactor. His formulations are often referred to[4-6] as the "variational perturbation theory" (VPT). Stacey considered[4,5] his VPT more general and more accurate than the GPT formulation of Usachev-Gandini (UG). Indeed, he showed[4,5] that the latter is a special case of VPT. Oblow,[7] on the other hand, has recently suggested that, physically, the Stacey VPT is a special case of the UG GPT; it is equivalent to a GPT formulation in which (a) the mechanism for maintaining criticality is the adjustment of the static eigenvalue (also referred to[7] as the k-reset mechanism), and (b) the alterations caused by this criticality-reset mechanism are allowed for, explicitly, in the perturbation expressions. The k-reset mechanism is physically equivalent to the adjustment of the average number of neutrons per fission. The purpose of this Letter is to clarify several questions concerning the relation between VPT and GPT.

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[2]A. GANDINI, *J. Nucl. Energy*, **21**, 755 (1967).

[3]W. M. STACEY, Jr., *J. Math. Phys.*, **13**, 1119 (1972).

[4]W. M. STACEY, Jr., *Nucl. Sci. Eng.*, **48**, 444 (1972).

[5]W. M. STACEY, Jr., *Variational Methods in Nuclear Reactor Physics*, Academic Press, New York (1974).

[6]H. HENRYSON, II et al., "Variational Sensitivity Analysis-Theory and Applications", *Proc. Mtg. Advanced Reactors; Physics, Design, and Economics* (Sep. 1974).

[7]E. M. OBLOW, "Reactor Cross-Section Sensitivity Studies Using Transport Theory," ORNL-TM-4436, Oak Ridge National Laboratory (1974).

Methods of Derivation

The first question considered is whether the VPT expressions can be derived only with variational techniques. The first evidence that this is not so was provided by Seki,[8] who derived, with the physical-consideration approach of UG, a GPT expression for the static reactivity for alterations that do not preserve criticality. Recently I have derived[9,10] VPT-like expressions for different types of integral parameters with conventional perturbation-theory techniques combined with equations for the flux difference and for the adjoint difference. Actually, Stacey[4] used the latter to show the connection between the generalized functions and the flux and adjoint perturbations. The evidence provided above leads to the conclusion that the VPT expressions of Stacey are not unique to the variational method. Hereafter I shall consider Stacey's expressions as one of the versions of GPT.

Criticality-Reset Mechanism and GPT

There are many mechanisms, either mathematical or physical, to restore criticality. To each of the criticality-reset mechanisms corresponds a version of GPT. The Stacey and the UG versions of GPT are two examples. In the UG formulation, the criticality-reset mechanism is assumed to be an implicit part of the system alteration. The Stacey formulation, on the other hand, uses k-reset, i.e., it adjusts the static eigenvalue to compensate for the alteration. An example of a third version of GPT is the GPT formulation in which criticality is maintained by the eigenvalue α reset.[11] In this version, the "time-absorption" eigenvalue (also the prompt-mode decay constant) is adjusted to preserve criticality. For illustration, three versions of GPT for reactivity are given here:

1. The implicit (i.e., UG) version of GPT for the static reactivity:

$$\rho_{\lambda I} = \rho_0 \left[1 - \langle \Gamma^+, (\delta \boldsymbol{A} - \lambda_0 \delta \boldsymbol{B}) \phi_0 \rangle\right] \quad . \tag{1}$$

2. The k-reset (i.e., Stacey) version for the same reactivity:

$$\rho_{\lambda V} = \rho_0 \left\{1 - \langle \Gamma_\lambda^+, [\delta \boldsymbol{A} - \delta(\lambda \boldsymbol{B})] \phi_0 \rangle\right\} \quad . \tag{2}$$

3. The α-reset version of GPT for the prompt-mode reactivity[12]:

$$\rho_\alpha = \rho_{\alpha 0} \left[1 - \left\langle \Gamma_\alpha^+, \left(-\frac{\delta\alpha}{v} + \delta \boldsymbol{A} - \delta \boldsymbol{B}_p\right) \phi_{\alpha 0} \right\rangle\right] \quad , \tag{3}$$

where

$$\rho_\alpha \equiv \frac{\langle \phi_0^+, (\delta \boldsymbol{A} - \delta \boldsymbol{B}) \phi_\alpha \rangle}{\langle \phi_0^+, \bar{\boldsymbol{B}} \phi_\alpha \rangle} \equiv \rho_\alpha(\phi_\alpha) \quad ,$$

$$\rho_{\alpha 0} \equiv \rho_\alpha(\phi_{\alpha 0}) \quad , \tag{4}$$

$$\left(-\frac{\alpha_0}{v} + \boldsymbol{A}_0 - \boldsymbol{B}_{p0}\right) \phi_{\alpha 0} = 0 \quad , \tag{5}$$

$$\rho_0 \equiv \frac{\langle \phi_0^+, (\delta \boldsymbol{A} - \lambda_0 \delta \boldsymbol{B}) \phi_0 \rangle}{\langle \phi_0^+, \boldsymbol{B} \phi_0 \rangle} \quad , \tag{6}$$

$$(\boldsymbol{A}_0 - \lambda_0 \boldsymbol{B}_0) \phi_0 = 0 \quad , \tag{7}$$

and

$$(\boldsymbol{A}_0^+ - \lambda_0 \boldsymbol{B}_0^+) \phi_0^+ = 0 \quad . \tag{8}$$

[8]Y. SEKI, *Nucl. Sci. Eng.*, **51**, 243 (1973).

[9]E. GREENSPAN, *Nucl. Sci. Eng.*, **56**, 107 (1975).

[10]E. GREENSPAN, *Trans. Israeli Nucl. Soc.*, **2**, 49 (1974).

[11]E. GREENSPAN, *Trans. Israeli Nucl. Soc.*, **2**, 56 (1974).

[12]E. GREENSPAN, *Nucl. Sci. Eng.*, **56**, 103 (1975).

The generalized functions are defined as follows:

$$(A_0^+ - \lambda_0 B_0^+)\,\Gamma^+ = \frac{(\overline{\delta A}^+ - \lambda_0 \overline{\delta B}^+)\,\phi_0^+}{\langle \phi_0^+, (\overline{\delta A} - \lambda_0 \overline{\delta B})\,\phi_0\rangle} - \frac{(B_0^+ + \overline{\delta B}^+)\,\phi_0^+}{\langle \phi_0^+, (B_0 + \overline{\delta B})\,\phi_0\rangle}\ , \tag{9}$$

$$(A_0^+ - \lambda_0 B_0^+)\,\Gamma_\lambda^+ = \frac{(\delta A^+ - \lambda_0 \delta B^+)\,\phi_0^+}{\langle \phi_0^+, (\delta A - \lambda_0 \delta B)\,\phi_0\rangle} - \frac{B^+\phi_0^+}{\langle \phi_0^+, B\,\phi_0\rangle}\ , \tag{10}$$

and

$$\left(-\frac{\alpha_0}{v} + A_0^+ - B_{p0}^+\right)\Gamma_\alpha^+ = \frac{(\delta A^+ - \delta B^+)\,\phi_0^+}{\langle \phi_0^+, (\delta A - \delta B)\,\phi_{\alpha 0}\rangle} - \frac{B^+\phi_0^+}{\langle \phi_0^+, B\phi_{\alpha 0}\rangle}\ . \tag{11}$$

The perturbation operators δA and δB pertain to the actual alterations in the reactor, whereas $\overline{\delta A}$ and $\overline{\delta B}$ take into account those alterations that result also from criticality reset. The function B_p is that part of the fission operator that takes into account the contribution of the prompt fission neutrons. It is concluded that the Stacey and UG versions are but two of many versions of GPT.

Applicability of Different Versions of GPT

The preceding discussion indicates that there is no generally preferred version of GPT. Each has its own range of applicability. The Stacey version is the right formulation for calculation of the effect of alterations on integral parameters that are functions of the static eigenvalue. Hence, it is not surprising that the Stacey formulation yields the static reactivity more accurately[5] than does the UG version. Similarly, the α-reset version of GPT is expected to be more accurate for calculating the effect of system alterations on such integral parameters as the prompt-mode reactivity and decay constant. Many system alterations encountered in the design and operation of nuclear reactors maintain criticality. For example, the change in the fuel composition due to burnup is compensated by a change in the concentration of burnable poisons. Uncertainties in input cross sections must be compensated in the design by changes in the composition or geometry of the reactor. The mechanism used to restore criticality can contribute significantly[13-15] to the effect of the alteration on different integral parameters. The UG version of GPT is the appropriate version for assessing the effect of those physical alterations that leave the reactor critical.

Terminology

It might be useful if a unified terminology were established for what is becoming an important field of perturbation theory. I propose that the term generalized perturbation theory be used for all perturbation-theory formulations in which the flux and adjoint perturbations are allowed for as correction factors that make first-order expressions correct to the second order. There are different versions of GPT, and these can be classified according to two categories: (a) the approach of allowing for the flux and adjoint perturbations, and (b) for homogeneous systems, the criticality-reset mechanism.

The perturbations in the distribution functions can be taken into account either in terms of generalized functions or in terms of perturbations in distribution functions. Equations (1), (2), and (3) provide examples for the generalized-function formulation. The same equations can also be expressed in terms of the flux alteration:

$$\rho_{\lambda I} = \rho_0 \left\{1 + \left\langle \delta\phi, \left[\frac{(\delta A^+ - \lambda_0 \delta B^+)\,\phi_0^+}{\langle \phi_0^+, (\delta A - \lambda_0 \delta B)\,\phi_0\rangle} - \frac{B^+\phi_0^+}{\langle \phi_0^+, B\,\phi_0\rangle}\right]\right\rangle\right\}\ , \tag{12}$$

$$\rho_{\lambda V} = \rho_0 \left\{1 + \left\langle \delta\phi_\lambda, \left[\frac{(\delta A^+ - \lambda_0 \delta B^+)\,\phi_0^+}{\langle \phi_0^+, (\delta A - \lambda_0 \delta B)\,\phi_0\rangle} - \frac{B^+\phi_0^+}{\langle \phi_0^+, B\,\phi_0\rangle}\right]\right\rangle\right\}\ , \tag{13}$$

and

$$\rho_\alpha = \rho_{\alpha 0} \left\{1 + \left\langle \delta\phi_\alpha, \left[\frac{(\delta A^+ - \delta B^+)\,\phi_0^+}{\langle \phi_0^+, (\delta A^+ - \delta B^+)\,\phi_{\alpha 0}\rangle} - \frac{B^+\phi_0^+}{\langle \phi_0^+, B\,\phi_{\alpha 0}\rangle}\right]\right\rangle\right\}\ , \tag{14}$$

where

$$(A_0 - \lambda_0 B_0)\,\delta\phi = -(\overline{\delta A} - \lambda_0 \overline{\delta B})\,\phi\ , \tag{15}$$

$$(A_0 - \lambda_0 B_0)\,\delta\phi_\lambda = -[\delta A - \delta(\lambda B)]\,\phi_\lambda\ , \tag{16}$$

and

$$\left(-\frac{\alpha_0}{v} + A_0 - B_{p0}\right)\delta\phi_\alpha = -\left(-\frac{\delta\alpha}{v} + \delta A - \delta B_p\right)\phi_\alpha\ . \tag{17}$$

In general, the generalized-function formulation is useful[9] for problems requiring the calculation of the effect of different system alterations on a given integral parameter. Conversely, the distribution alteration is the useful approach for problems requiring the calculation of the effect of a given system alteration on different integral parameters.

Each of the GPT formulations should also be classified according to the criticality-reset mechanism. For example, Eq. (13) is referred to, in the terminology proposed, as the k-reset version of GPT for the static reactivity expressed in terms of the flux alteration. Similary Eq. (3) is the α-reset version of GPT for the prompt-mode reactivity expressed in terms of generalized functions.

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[14]A. GANDINI and M. SALVATORES, *Nucl. Sci. Eng.*, **41**, 452 (1970).

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The Streaming Term of the Transport Equation in Terms of General Orthogonal Coordinates

From time to time, papers appear that suggest that the evaluation of the streaming term in the transport equation is a complicated and laborious process when the coordinate system is not Cartesian. (See, for example, Ref. 1.) In fact, it is easy to do the calculation in a compact manner. Perhaps everyone knows the scheme I shall describe. However, although I have used it for some time in teaching, I know of no reference in which it is easily available. Perhaps, for this reason, I may be excused for presenting what might be common knowledge.

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