

represented by a change of a cross section,  $\sigma_k$ , the sensitivity of the breeding ratio,  $BR$ , to it will be

$$\frac{d(BR)}{d\sigma_k} = \frac{\partial(BR)}{\partial\sigma_k} + \frac{\partial(BR)}{\partial P_c} \frac{\partial P_c}{\partial\sigma_k}, \quad (1)$$

where  $P_c$  represents the parameter chosen for reestablishing criticality.

2. *Cross-section adjustment.* As is well known, this represents an important and wide application of the generalized perturbation methods, since they allow the calculation of the sensitivities of the various integral parameters to the cross sections. With these adjustments, the cross sections are forced to become statistically consistent with a variety of integral parameters: reaction rate ratios, reactivity worths, prompt neutron lifetimes, etc. An important parameter that obviously should be included is represented by the (measured) system reactivity, in the sense that the perturbations inherent to all the cross-section adjustments should total a zero contribution. In fact, all these measurements were made on *critical* facilities and, therefore, all the cross-section changes should be forced so as to maintain criticality, within the experimental errors, if the adjusted values are to be consistent with the experimental evidence.

3. *Reactivity worths.* In this case, the generalized perturbation methods can successfully be applied to evaluate changes induced in a reactor system by an alteration  $\delta P$  affecting a reactivity worth, as given by the ratio

$$\rho = \frac{\langle \phi^* \Delta P \phi' \rangle}{\langle \phi^* F' \phi' \rangle}, \quad (2)$$

without being forced to recalculate  $\phi'$  for each altered system<sup>14</sup> [easily calculable direct effects of the perturbation  $\delta P$  on  $\Delta P$  or on  $F'$  of Eq. (2) are not considered here]. Here again we meet the requirement of maintaining criticality. In fact, rather than the reactivity value itself, the designer needs ultimately to know, in an accident analysis, the evolution of a given sequence of events in a particular unaltered system and the evolution of the same sequence after alterations (of temperature, composition, etc.) have been introduced. So that the comparison among these cases has sense, the sequence of events and the starting conditions must be the same.<sup>15</sup> Therefore, after evaluating a given sequence of events (for instance a sodium voiding) in an unaltered system, evaluation should be made of the same sequence in the system affected by a given alteration (with respect to temperature, fuel composition, etc.) recognizing the requirement that such alteration maintain criticality under steady state conditions (i.e., at times immediately preceding the initiation of the sequence itself). Merely evaluating the effect on the reactivity of a sodium void by, say, a different fission cross section of <sup>239</sup>Pu does not, in principle, make much sense if we do not give due consideration to the fact that such a different cross section implies itself an altered critical system (for instance, with different fuel enrichment or size to maintain criticality). Such alterations should then also be included in the perturbation to give to the reactor designer a proper value of the sodium worth.

<sup>14</sup>More precisely, these generalized perturbation methods give an estimate corresponding to a change  $\delta\phi$  rather than  $\delta\phi'$  with  $\phi'$  of Eq. (2) replaced by  $\phi$ . [The change  $\Delta\phi = (\phi' - \phi)$ , due to the self-perturbation effect, may have been accounted for separately by the same methods, as previously described.] This amounts to neglecting second-order effects on the flux.

<sup>15</sup>Apart, of course, from the alteration itself.

4. *Reaction rate ratios.* This case is similar to those discussed above and the conclusions are identical. These measurements are made on critical reactors, and if we need to know the effect of changes on their calculated values resulting from system alterations, these should, in any case, not alter the criticality of the system.

All the examples suggested in Ref. 1 for application of these perturbation methods fall within the above-described cases. To further clarify this important point, consider again, more closely, the relevant case of the breeding ratio. In this event the character of the adjustment necessary to reestablish criticality can significantly change the results.<sup>16</sup> If, for example, the design implies that a different fuel enrichment should be specified in case criticality was badly calculated because of, say, a rather inaccurate plutonium fission cross section, the impact on the breeding ratio of changing such a parameter (in a project analysis survey) will be quite different than in the case where a core size change is foreseen in the same circumstance. In fact, an enrichment change would imply, above all, a strong direct effect on the internal breeding ratio, the ratio of fissile to fertile materials in the core involved. A size change would imply mostly changing the respective contributions from the internal and external breeding ratios to the total one.

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<sup>16</sup>Many practical survey studies are made by theoreticians without a particular reactor project in mind for which an assigned criticality readjustment is specified on technical or economical bases. The analysis can be of an unidentified conceptual reference system and the readjustment can become problematical. In these cases one should assume a set of reasonable hypotheses and consider all of them in the analysis. An approach of this kind was followed, for example, in Refs. 17 and 18 in relation to the breeding ratio.

<sup>17</sup>A. GANDINI, "Study of the Sensitivity of Calculations for Fast Reactors Fueled with Pu<sup>239</sup>-U<sup>238</sup> and U<sup>233</sup>-Th to Uncertainties in Nuclear Data," ANL-6608, Argonne National Laboratory (1962).

<sup>18</sup>A. GANDINI, M. SALVATORES, G. SENA, and I. DAL BONO, "Analysis of Fast Reactors by the CIAP and GLOBPERT Codes Using Improved Perturbation Methods," p. 304, ANL-7320, Argonne National Laboratory (1966).

#### Response to "Comments on Variational Theory and Generalized Perturbation Methods"

Mr. Gandini argues that it is appropriate in perturbation theory to use a formalism in which the eigenvalue is unchanged because a compensating perturbation must be made to maintain criticality. However, the appropriate formalism depends on just what question is being posed. Mr. Gandini gives several examples of one type of question—if one has a fixed reference case, has good reason to believe his reference calculation is correct, and wants to know the effect of some physical change that would require compensation, then it is appropriate to use a formalism in which the net reactivity worth of the perturbation plus compensation is zero. In this case, the  $\delta k$  terms could be omitted in the variational formalism, or they could be

retained and allowed to cancel. On the other hand, one may wish to know how much difference there is in some integral parameter due to two different ways of doing the reactor calculation. For example, one may wish to calculate the sodium worth in a critical facility with two different cross-section sets, neither of which predicts the critical mass correctly. The straightforward procedure would be to make a separate flux calculation with each cross-section set (obtaining different eigenvalues), calculate the sodium worth in each case, and subtract the two. The variational procedure, with the  $\delta k$  correction, would be appropriate in this case. Thus, the appropriate formalism in any particular case depends on just how the question is put, and the variational formalism seems to have sufficient generality to accommodate a variety of questions.

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### Variational Versus Generalized Perturbation Theories—Are They Different?

The number and scope of applications of perturbation-theory formulations for integral parameters of the form of ratios of linear and bilinear functionals has greatly increased in recent years. There are several versions of these formulations that differ in the form of the perturbation expressions, in the approaches used for deriving these expressions, and in the terminology used to refer to them.

Usachev<sup>1</sup> and Gandini<sup>2</sup> have derived a generalized perturbation theory (GPT) on the basis of physical considerations. Their GPT formulations are restricted to alterations that leave the reactor critical. Using variational methods, Stacey<sup>3,4</sup> derived similar expressions that allow for alterations that change the static eigenvalue of the reactor. His formulations are often referred to<sup>4-6</sup> as the "variational perturbation theory" (VPT). Stacey considered<sup>4,5</sup> his VPT more general and more accurate than the GPT formulation of Usachev-Gandini (UG). Indeed, he showed<sup>4,5</sup> that the latter is a special case of VPT. Oblov,<sup>7</sup> on the other hand, has recently suggested that, physically, the Stacey VPT is a special case of the UG GPT; it is equivalent to a GPT formulation in which (a) the mechanism for maintaining criticality is the adjustment of the static eigenvalue (also referred to<sup>7</sup> as the  $k$ -reset mechanism), and (b) the alterations caused by this criticality-reset mechanism are allowed for, explicitly, in the perturbation expressions. The  $k$ -reset mechanism is physically equivalent to the adjustment of the average number of neutrons per fission. The purpose of this Letter is to clarify several questions concerning the relation between VPT and GPT.

<sup>1</sup>L. N. USACHEV, *J. Nucl. Energy, Part A/B*, **18**, 571 (1966).

<sup>2</sup>A. GANDINI, *J. Nucl. Energy*, **21**, 755 (1967).

<sup>3</sup>W. M. STACEY, Jr., *J. Math. Phys.*, **13**, 1119 (1972).

<sup>4</sup>W. M. STACEY, Jr., *Nucl. Sci. Eng.*, **48**, 444 (1972).

<sup>5</sup>W. M. STACEY, Jr., *Variational Methods in Nuclear Reactor Physics*, Academic Press, New York (1974).

<sup>6</sup>H. HENRYSON, II et al., "Variational Sensitivity Analysis—Theory and Applications", *Proc. Mtg. Advanced Reactors; Physics, Design, and Economics* (Sep. 1974).

<sup>7</sup>E. M. OBLow, "Reactor Cross-Section Sensitivity Studies Using Transport Theory," ORNL-TM-4436, Oak Ridge National Laboratory (1974).

### Methods of Derivation

The first question considered is whether the VPT expressions can be derived only with variational techniques. The first evidence that this is not so was provided by Seki,<sup>8</sup> who derived, with the physical-consideration approach of UG, a GPT expression for the static reactivity for alterations that do not preserve criticality. Recently I have derived<sup>9,10</sup> VPT-like expressions for different types of integral parameters with conventional perturbation-theory techniques combined with equations for the flux difference and for the adjoint difference. Actually, Stacey<sup>4</sup> used the latter to show the connection between the generalized functions and the flux and adjoint perturbations. The evidence provided above leads to the conclusion that the VPT expressions of Stacey are not unique to the variational method. Hereafter I shall consider Stacey's expressions as one of the versions of GPT.

### Criticality-Reset Mechanism and GPT

There are many mechanisms, either mathematical or physical, to restore criticality. To each of the criticality-reset mechanisms corresponds a version of GPT. The Stacey and the UG versions of GPT are two examples. In the UG formulation, the criticality-reset mechanism is assumed to be an implicit part of the system alteration. The Stacey formulation, on the other hand, uses  $k$ -reset, i.e., it adjusts the static eigenvalue to compensate for the alteration. An example of a third version of GPT is the GPT formulation in which criticality is maintained by the eigenvalue  $\alpha$  reset.<sup>11</sup> In this version, the "time-absorption" eigenvalue (also the prompt-mode decay constant) is adjusted to preserve criticality. For illustration, three versions of GPT for reactivity are given here:

1. The implicit (i.e., UG) version of GPT for the static reactivity:

$$\rho_{\lambda I} = \rho_0 [1 - \langle \Gamma^+, (\delta \mathbf{A} - \lambda_0 \delta \mathbf{B}) \phi_0 \rangle] . \quad (1)$$

2. The  $k$ -reset (i.e., Stacey) version for the same reactivity:

$$\rho_{\lambda V} = \rho_0 \{1 - \langle \Gamma_\lambda^+, [\delta \mathbf{A} - \delta (\lambda \mathbf{B})] \phi_0 \rangle\} . \quad (2)$$

3. The  $\alpha$ -reset version of GPT for the prompt-mode reactivity<sup>12</sup>:

$$\rho_\alpha = \rho_{\alpha 0} \left[ 1 - \left\langle \Gamma_\alpha^+, \left( -\frac{\delta \alpha}{v} + \delta \mathbf{A} - \delta \mathbf{B}_p \right) \phi_{\alpha 0} \right\rangle \right] , \quad (3)$$

where

$$\rho_\alpha \equiv \frac{\langle \phi_0^+, (\delta \mathbf{A} - \delta \mathbf{B}) \phi_\alpha \rangle}{\langle \phi_0^+, \bar{\mathbf{B}} \phi_\alpha \rangle} \equiv \rho_\alpha(\phi_\alpha) ,$$

$$\rho_{\alpha 0} \equiv \rho_\alpha(\phi_{\alpha 0}) , \quad (4)$$

$$\left( -\frac{\alpha_0}{v} + \mathbf{A}_0 - \mathbf{B}_{p0} \right) \phi_{\alpha 0} = \mathbf{0} , \quad (5)$$

$$\rho_0 \equiv \frac{\langle \phi_0^+, (\delta \mathbf{A} - \lambda_0 \delta \mathbf{B}) \phi_0 \rangle}{\langle \phi_0^+, \bar{\mathbf{B}} \phi_0 \rangle} , \quad (6)$$

$$(\mathbf{A}_0 - \lambda_0 \mathbf{B}_0) \phi_0 = \mathbf{0} , \quad (7)$$

and

$$(\mathbf{A}_0^+ - \lambda_0 \mathbf{B}_0^+) \phi_0^+ = \mathbf{0} . \quad (8)$$

<sup>8</sup>Y. SEKI, *Nucl. Sci. Eng.*, **51**, 243 (1973).

<sup>9</sup>E. GREENSPAN, *Nucl. Sci. Eng.*, **56**, 107 (1975).

<sup>10</sup>E. GREENSPAN, *Trans. Israeli Nucl. Soc.*, **2**, 49 (1974).

<sup>11</sup>E. GREENSPAN, *Trans. Israeli Nucl. Soc.*, **2**, 56 (1974).

<sup>12</sup>E. GREENSPAN, *Nucl. Sci. Eng.*, **56**, 103 (1975).