Now let[8]

$$m_n(u) \triangleq \left[e^u - \sum_{R=0}^{n-1} \frac{u^R}{R!}\right] H(u) = \mathcal{L}^{-1}\left[(s-1)^{-1} s^{-n}\right] ,$$

$$n = 1, 2, \ldots \quad . \qquad (19)$$

Then, making use of the definition in Eq. (4), one finds[8]

$$\mathcal{L}^{-1}\left[s^{-1}\bar{f}^n(s)\right] = (1-\alpha)^{-n} e^{-u} \mathcal{L}^{-1}\{(s-1)^{-1} s^{-n} \times [1 - \exp(-s\beta)]^n\}$$

$$= (1-\alpha)^{-n} e^{-u} \sum_{k=0}^{n} \mathcal{L}^{-1}\left[\binom{n}{k}(-1)^k \exp(-s\beta k)(s-1)^{-1} s^{-n}\right]$$

$$= (1-\alpha)^{-n} e^{-u} \sum_{k=0}^{n} (-1)^k \binom{n}{k} H(u-\beta k)\, m_n(u-\beta k) \qquad (20)$$

for $n = 1, 2, \ldots$. Substitution of Eq. (20) into Eq. (18) yields the value of $P[M(u) = n]$. The result coincides with Barnett's[1] Eq. (53).

Making use of Eq. (18) we find

$$E[M(u)] \triangleq \sum_{n=1}^{\infty} n P[M(u) = n]$$

$$= \mathcal{L}^{-1}\left\{\sum_{n=1}^{\infty} n s^{-1}[\bar{f}^{n-1}(s) - \bar{f}^n(s)]\right\}$$

$$= \mathcal{L}^{-1}\{s^{-1}[1 - \bar{f}(s)]^{-1}\} \qquad (21a)$$

$$E[M^2(u)] \triangleq \sum_{n=1}^{\infty} n^2 P[M(u) = n]$$

$$= \mathcal{L}^{-1}\left\{\sum_{n=1}^{\infty} n^2 s^{-1}[\bar{f}^{n-1}(s) - \bar{f}^n(s)]\right\}$$

$$= \mathcal{L}^{-1}\left[s^{-1}\sum_{n=0}^{\infty}(2n+1)\bar{f}^n(s)\right]$$

$$= \mathcal{L}^{-1}\{s^{-1}[1 + \bar{f}(s)][1 - \bar{f}(s)]^{-2}\} \quad . \qquad (21b)$$

Equation (21) shows that $E[M(u)]$ is the integral with respect to u of the function $\chi(u)$, which is discussed in Sec. II of Ref. 7. Now we observe that $[1 - \bar{f}(0)] = 0$. Furthermore, if we let $x = Re(s)$ and $\omega = Im(s)$, we find

$$|\bar{f}(x + i\omega)| \le \int_0^\infty du \exp(-xu) f(u) < 1 , \quad \text{for} \quad x > 0$$

$$|Re\,\bar{f}(i\omega)| = \left|\int_0^\infty du \cos(\omega u) f(u)\right| < 1 , \quad \text{for} \quad \omega \neq 0 .$$

Hence, in the complex plane, the origin is the rightmost singularity[9] of $[1 - \bar{f}(s)]^{-1}$. Now, known theorems[10] link the behavior of the transformed function $\bar{g}(s)$ in the neighborhood of its rightmost singularity with the behavior of $g(u) = \mathcal{L}^{-1}[\bar{g}(.)]$ for $u \to +\infty$. Employing definition (7) we find

$$s^{-1}[1 - \bar{f}(s)]^{-1} = s^{-2} \cdot \frac{-1}{f^{(1)}} + \frac{s^{-1}}{2} \frac{f^{(2)}}{[f^{(1)}]^2} + O(1) ,$$

$$\text{for } s \to 0 \qquad (22a)$$

$$s^{-1}[1 + f(s)][1 - f(s)]^{-2}$$

$$= \frac{1}{[f^{(1)}]^2}\left(2s^{-3} + s^{-2}\left[f^{(1)} - 2\frac{f^{(2)}}{f^{(1)}}\right] + s^{-1} \times \left\{\frac{3}{2}\left[\frac{f^{(2)}}{f^{(1)}}\right]^2 - \frac{2}{3}\frac{f^{(3)}}{f^{(1)}} - \frac{f^{(2)}}{2}\right\}\right) + O(1) , \quad \text{for } s \to 0 . \qquad (22b)$$

Then[10] we are entitled to claim that

$$E[M(u)] = \frac{u}{E[w]} + \frac{1}{2}\frac{E[w^2]}{(E[w])^2} + o(u^{-1}) , \quad \text{for} \quad u \to +\infty \qquad (23a)$$

$$E[M^2(u)] = \left(\frac{u}{E[w]}\right)^2 + \frac{u}{E[w]}\left\{2\frac{E[w^2]}{(E[w])^2} - 1\right\} + \left\{\frac{3}{2}\frac{(E[w^2])^2}{(E[w])^4} - \frac{2}{3}\frac{E[w^3]}{(E[w])^3} - \frac{1}{2}\frac{E[w^2]}{(E[w])^2}\right\} + o(u^{-1}) , \quad \text{for} \quad u \to +\infty , \qquad (23b)$$

where Eqs. (8) were employed. Result (23a) yields Barnett's[1] Eq. (43). Moreover, by combining Eqs. (23a) and (23b) and after some manipulations, Barnett's[1] result (54) can be obtained.

SOME CONCLUSIONS

Classical transport theory has been employed to derive results that Barnett[1] obtained by a probability-theoretical method; thus, the scope of classical transport theory has been shown not to be limited to the evaluation of average behaviors.

The transport theory approach seems to involve simpler mathematical procedures. However, it must be recognized that results given in Eq. (23) have been obtained here by a mathematical procedure that is only heuristic. A rigorous treatment would require proof that the series and the inverse transformation appearing in Eqs. (21a) and (21b) commute. Furthermore, the transition from Eqs. (22a) and (22b)—where the O symbol appears—to Eqs. (23a) and (23b)—where the o symbol appears—ought to be justified by checking to see that all assumptions of the relevant theorems[10] are satisfied.

S. L. Paveri-Fontana

Instituto di Meccanica Razionale
Università di Bari
Via Nicolai, 2
70121 Bari, Italy

December 9, 1974

ACKNOWLEDGMENT

This work was performed under the auspices of CNR, Italy.

[8]F. OBERHETTINGER and L. BADII, *Tables of Laplace Transforms*, Springer-Verlag, New York (1973).

[9]The location of the real roots of $[1 - \bar{f}(s)] = 0$ has been studied recently by C. E. SIEWERT and A. R. BURKARDT, *Nucl. Sci. Eng.*, **54**, 455 and **55**, 247 (1974).

[10]G. DOETSCH, *Handbuch der Laplace Transformation*, Vols. I and II, Verlag Birkhauser, Basel (1950 and 1956).

Response to "Comments on 'On the Randomness of a Neutron's Kinetic Energy as It Slows Down by Elastic Collisions in an Infinite Medium' "

It is, of course, rewarding to see the results confirmed by a different method, particularly when that method is more familiar to readers of *Nuclear Science and Engineer-*

ing than the one employed in the Note. As for the opinion ". . . it appears that the interesting features which are inherent in establishing a connection between transport theory and probability theory can be somewhat hindered by the computational intricacies . . . ," I prefer not to become involved in a "my method can whip your method" debate. Isn't it true that whether or not a method, concept, or procedure is considered simple depends at least as strongly upon the perceiver as upon the perceived? It seems to me that it is desirable for the practitioner to have as many tools available as possible.

As a final note, unrelated to Paveri-Fontana's comments, allow me to correct an error in my Note. I've thought further about the last paragraph concerning absorbing media. The paragraph is nonsense.

Charles S. Barnett

Lawrence Livermore Laboratory
P.O. Box 808
Livermore, California 94550

December 23, 1974