the neutron density and reactivity respectively

$$\frac{n}{\delta k} = \frac{1}{KTC}\left(\frac{1}{G}\frac{KTCK_R\,GG_R}{1 + KTCK_R\,GG_R}\right) \tag{10}$$

Since the terms in the brackets describe the dynamic behavior of the system, only they need be examined. Plots of the attenuation frequency and phase-shift frequency curves of these transfer functions are shown in Figs. 6–9, when the open-loop gain is equal to $KTC\ K_R = 54.8$ db and 44.8 db.

The above comparison between the distributed and lumped transfer functions was done on the basis of $\lambda = \infty$. To show the error that arises due to this assumption, the open-loop transfer functions for three models are plotted in Fig. 10 for the case $w = 2.4$ meters/sec. If we compare these curves with those in Fig. 4 we see that the assumption $\lambda = \infty$ makes the open-loop gain coefficient smaller than if $\lambda \neq \infty$, when in both cases the closed loop gain is equal to $M = 2.28$ db.

The results that have been presented above give a view of the significance of the simplifying assumptions made in order to simulate the effect of the distributed parameters. As shown by the numerical calculations, these assumptions cause some differences in dynamic performance of a reactor temperature feedback loop. These differences are dependent on the parameters such as coolant velocity, effective time between succeeding generations of neutrons, and open loop gain ($KTC\ K_R$). For $KTC\ K_R \leqq 44.8$, when the neutron density is a controlled variable, one section may be adequate to obtain good accuracy (Fig. 8). For $KTC\ K_R \gtrsim 54.8$, or for the case when the coolant temperature is the controlled variable, use of six reactor sections may not be satisfactory (see Figs. 6–9).

The final conclusion is that the distributed model of the thermal processes in the reactor core allows the open loop gain coefficient to be greater in comparison with the lumped model at the same conditions of damping of the closed loop. By this we mean, the lumped approximation being more instable predicts the limiting value of M_p at a lower value of the gain coefficient than does the distributed parameter system.

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Received March 23, 1962
Revised April 30, 1963

The Fuchs–Nordheim Model with Variable Heat Capacity

In this letter we shall describe how the values of certain quantities of physical interest, such as final fuel element temperature, peak power, etc., may be computed for a pulsed reactor with variable heat capacity.

For many cases of practical importance, the heat capacity, C, of the reactor fuel elements may be assumed to vary linearly with temperature, T:

$$C = C_0 + \gamma T \tag{1}$$

where C_0 and γ are constants. The so-called Fuchs–Nordheim model (*1*) yields for the equations of motion of the reactor

$$\frac{dP}{dt} = \frac{\delta k_p - \alpha T}{l} P \tag{2}$$

$$C\frac{dT}{dt} = P \tag{3}$$

where $P(t)$ is the reactor power at time t, δk_p is the prompt reactivity insert, α is the magnitude of the prompt reactivity temperature coefficient (assumed constant), and l is the prompt neutron lifetime. In (2) and (3) delayed neutron and heat transfer effects are neglected; for narrow pulses, such as are obtained in the TRIGA reactor, these are excellent approximations. The neglect of space and neutron energy dependent effects is also a good approximation and this question will be examined elsewhere (*2*).

With the variation (1) for the heat capacity of the fuel elements the equations of motion may be integrated analytically with the result that important quantities, such as total fuel element temperature rise, peak power, and total energy release, are expressible concisely as functions of a single dimensionless parameter,

$$\sigma = \alpha C_0/\gamma(\delta k_p). \tag{4}$$

This may be seen most easily by introducing the dimensionless variables

$$x \equiv t/\tau \tag{5}$$

$$Q \equiv [\alpha l/C_0(\delta k_p)^2]P \tag{6}$$

$$\theta \equiv (\alpha/\delta k_p)T \tag{7}$$

where the asymptotic reactor relaxation time,

$$\tau = l/\delta k_p\,. \tag{8}$$

Then Eqs. (2) and (3) become

$$dQ/dx = (1 - \theta)Q \tag{9}$$

$$d\theta/dx = \sigma Q/(\theta + \sigma). \tag{10}$$

Division of (9) by (10) and integration yields for the relation between power and temperature

$$Q - Q_0 = \theta + \frac{(1-\sigma)\theta^2}{2\sigma} - \frac{\theta^3}{3\sigma} \tag{11}$$

where Q_0 is the initial value of the power, in the above defined units. The value of the final temperature, T_∞, may be obtained to an excellent approximation by setting $Q = Q_0$ in (11), since the peak power is generally so much greater than Q_0. The useful result is

$$T_\infty = \frac{\delta k_p}{\alpha}\left[-\tfrac{3}{4}(\sigma - 1) + \tfrac{3}{4}\sqrt{(\sigma-1)^2 + \tfrac{16}{3}\sigma}\right]. \tag{12}$$

This reduces to

$$T_\infty = 2\delta k_p/\alpha \quad \text{for} \quad \sigma = \infty \tag{13}$$

which is the usual result for constant heat capacity ($\gamma = 0$), and to

$$T_\infty = \tfrac{3}{2}\delta k_p/\alpha \quad \text{for} \quad \sigma = 0 \tag{14}$$

so that a 25% reduction in final fuel element temperature is possible for this case. The total energy release may be computed from (12) by noting that

$$E_\infty = C_0 T_\infty + (\gamma T_\infty^2/2). \tag{15}$$

The peak power occurs at $\theta = 1$, as is clear from (9), so that (11) yields

$$P_{\max} - P_0 = \frac{1 + 3\sigma}{6\sigma}\frac{C_0(\delta k_p)^2}{\alpha l}. \tag{16}$$

It is of interest to discuss the physical interpretation of the various limiting cases here also but, for brevity, we shall compare (16) with the constant heat capacity result

$$P_{\max}^{(F)} - P_0 = \frac{\bar{C}(\delta k_p)^2}{2\alpha l} = \frac{C_0(\delta k_p)^2}{2\alpha l}\frac{1 + \alpha}{\sigma} \tag{17}$$

where we have chosen an "average" heat capacity over the course of the pulse

$$\bar{C} = C_0 + \gamma\left(\frac{\delta k_p}{\alpha}\right) = C_0\left(1 + \frac{1}{\sigma}\right). \tag{18}$$

From (16) and (17)

$$\frac{P_{\max} - P_0}{P_{\max}^{(F)} - P_0} = \frac{1 + 3\sigma}{3(1 + \sigma)} \tag{19}$$

so that (neglecting P_0)

$$P_{\max}/P_{\max}^{(F)} = 1 \quad \text{for} \quad \sigma = \infty \tag{20}$$

as expected and

$$P_{\max}/P_{\max}^{(F)} = \tfrac{1}{3} \quad \text{for} \quad \sigma = 0, \tag{21}$$

i.e., a drop of 67% is possible with respect to the value obtained by using an average heat capacity. Needless to say, values for specific cases should be computed using the full formulas (12) and (16), and due regard must be paid to the validity of the linear approximation (1).

We may finally remark that substitution of (11) into (10) and a decomposition into partial fractions enables one to carry out the final integration which gives the temperature as a function of time; since the dependence is implicit, i.e., $f(T) = t$, where f is a transcendental function, we shall forego a detailed discussion (*3*). However, it may be noted that a few percent broadening of the pulse width, as compared to the constant heat capacity case, is a general characteristic of the solution.

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Received April 2, 1963

Green's Function for a Bare Slab with Anisotropic Scattering*

Under the assumption that the scattering function can be expanded into a finite series of legendre polynomials, the one-velocity Boltzmann equation in the case of plane symmetry has the form

$$\mu\frac{\partial\psi(x,\mu)}{\partial x} + \psi(x,\mu) = \frac{c}{2}\sum_{k=0}^{N} b_k P_k(\mu)\int_{-1}^{1} P_k(\mu')\psi(x,\mu')\,d\mu' \tag{1}$$

where x is in terms of optical thickness (*1*), c is the mean number of secondarys which emanate from a neutron-nucleus interaction, and the b_k are the coefficients of the Legendre polynomial expansion. The general solution to this equation has been found by A. Jacobs (*2*) and J. Mika (*3*) in the form

$$\psi(x,\mu) = \sum_{j=1}^{M} a_{+j}\phi(+L_j,\mu)e^{-x/L_j} + \sum_{j=1}^{M} a_{-j}\phi(-L_j,\mu)e^{x/L_j} + \int_{-1}^{1} A(\nu)\phi(\nu,\mu)e^{-x/\nu}\,d\nu \tag{2}$$

where the ϕ's are known functions and the coefficients $a_{\pm j}$ and $A(\nu)$ are determined from boundary conditions. The eigenfunctions have useful orthogonality properties and have been shown complete in the space of prescribed boundary variations, $\psi(\mu)$, which satisfy Eq. (1).

The Green's function problem for a bare slab of thickness T with a source plane at x_0 emitting neutrons at $\mu = \mu_0$ can be defined as follows

$$\psi^+(T,\mu) = 0 \qquad \mu < 0 \tag{3}$$

$$\psi^-(0,\mu) = 0 \qquad \mu > 0 \tag{4}$$

$$\mu[\psi^+(x_0,\mu) - \psi^-(x_0,\mu)] = \delta(\mu - \mu_0) \tag{5}$$

The quantities ψ^+ and ψ^- refer to the neutron distributions to the right and left of the source plane respectively.

Due to the nature of the boundary conditions, in the calculations that follow there will occur coefficients which are zero over half their respective ranges in ν. It will therefore be convenient to decompose the continuous coefficients in the eigenfunction expansion (2), into two half-range coefficients (*4*). Putting the discrete summations in a more compact notation, Eq. (2) may be written as

$$\psi^\pm(x,\mu) = \sum_{\pm L_j}^{M} a_{\pm j}^{\pm}\phi(\pm L_j,\mu)e^{\mp x/L_j} + \int_{-1}^{1}[A^\pm(\nu)h(\nu) + B^\pm(-\nu)h(-\nu)e^{b^\pm/\nu}]\phi(\nu,\mu)e^{-x/\nu}\,d\nu \tag{6}$$

where $h(\nu)$is the Heaviside step function and where

$$b^+ = T; \qquad b^- = x_0$$

Upon application of the orthogonality properties of the eigenfunctions Eq. (5) yields

$$A^+(\nu) - A^-(\nu) = \phi(\nu,\mu_0)e^{x_0/\nu}/M(\nu) \quad \nu > 0 \tag{7a}$$

$$B^+(\nu)e^{(x_0-T)/\nu} - B^-(\nu) = \phi(-\nu,\mu_0)/M(-\nu) \quad \nu > 0 \tag{7b}$$

$$a_{\pm j}^{+} - a_{\pm j}^{-} = \frac{\phi(\pm L_j,\mu_0)}{M_{\pm j}}e^{\pm x_0/L_j} \tag{7c}$$

* This work was supported by a grant from the National Science Foundation.