# Letters to the Editors

## Xenon Oscillations in Finite Reactors

The modal theory of xenon oscillations (*1*, *2*) in large thermal reactors makes it possible to calculate approximately the threshold flux at which free (neither growing nor dying) xenon oscillations occur. Randall and St. John (*2*) have derived the general condition (1) given below which holds at the threshold of oscillations in a reactor, and have applied the condition (1) to the special case of a one-dimensional slab reactor. (A "one-dimensional slab reactor" is defined as having constant flux in the directions at right angles to the thickness of the slab.) In this letter the general condition (1) is applied to a bare parallelepiped and it is found that the threshold for oscillation is different from the threshold of oscillation in a one-dimensional slab of the same thickness. Thus the use of the one-dimensional slab model leads to errors in the prediction of oscillation thresholds in three-dimensional reactors.

Neglecting the effect of temperature on reactivity and the direct yield of xenon, the condition at the oscillation threshold is (*2*)

$$\mu_1^2 = -a\bar{x}_1 \Big/ \left(1 + \frac{\lambda_i + \lambda_x}{\sigma\bar{\phi}_1}\right) \tag{1}$$

where $\lambda_i$ and $\lambda_x$ are the decay constants of iodine-135 and xenon-135 respectively, $\sigma$ is the cross section of xenon-135, $a$ is a reactor constant,

$$\bar{x}_1 = \int_{\text{pile}} g_1^2 \times dV$$

where $x$ is the unperturbed concentration of xenon in the reactor,

$$\bar{\phi}_1 = \int_{\text{pile}} g_1^2\phi \, dV \tag{2}$$

and $\phi$ is the unperturbed flux. $g_1$ is a normalized eigenfunction, with eigenvalue $\mu_1^2$, and is the solution to the equation

$$\nabla^2 g_1 + B^2 g_1 + \mu_1^2 g_1 = 0$$

which has the same outer boundary conditions as the flux. $B^2$ is the material buckling. As in ref. *2* the mode which we shall discuss in the slab has a single internal nodal plane parallel to the side of the slab.

We shall now compare the threshold flux of a uniform one-dimensional slab of extrapolated thickness $X$ to that of a uniform parallelepiped of the same extrapolated thickness. In both cases $\mu_1^2 = 3\pi^2/X^2$. The threshold value of $\bar{\phi}_1$ in Eq. (1) is a function of $X$, of reactor constants, and of $\bar{x}_1$. $\bar{x}_1$ is only a slowly varying function of flux above fluxes of $5 \times 10^{13}$. It follows that the threshold value of $\bar{\phi}_1$ is approximately the same in a parallelepiped as in a "one-dimensional slab" of equal extrapolated thickness $X$. However, because of the different geometries it is seen below that the threshold values of the maximum in the flux distribution are different in the two cases, and that the threshold values of the average fluxes are different.

In the case of a one-dimensional slab of uniform composition

$$\phi = \phi_{\max}(\text{slab}) \sin (\pi x/X)$$

and

$$g_1 = \sqrt{2/X} \sin (2\pi x/X)$$

where $\int g_1^2 \, dx = 1$. Substituting in Eq. (2) we obtain

$$\bar{\phi}_1 = 0.679 \ \phi_{\max}(\text{slab}) \text{ or } 1.067 \ \phi_{\text{av}}(\text{slab}) \tag{3}$$

where $\phi_{\text{av}}(\text{slab}) = (2/\pi)\phi_{\max}(\text{slab})$, which is the result obtained previously (*2*).

In the case of a parallelepiped of uniform composition

$$\phi = \phi_{\max}(\text{par.}) \sin (\pi x/X) \cdot \sin (\pi y/Y) \cdot \sin (\pi z/Z),$$

and

$$g_1 = \sqrt{2/X} \sin (2\pi x/X) \cdot \sqrt{2/Y} \sin (\pi y/Y) \cdot \sqrt{2/Z} \sin (\pi z/Z)$$

where $\int g_1^2 \, dV = 1$. Substituting in Eq. (2) we obtain

$$\bar{\phi}_1 = 0.489 \ \phi_{\max}(\text{par.}) \text{ or } 1.896 \ \phi_{\text{av}}(\text{par.}) \tag{4}$$

where

$$\phi_{\text{av}}(\text{par.}) = (2/\pi)^3\phi_{\max}(\text{par.}).$$

Equations (3) and (4) show that although the slab and parallelepiped may have approximately equal threshold values of $\bar{\phi}_1$, the threshold fluxes are different. For instance the threshold of the average value of the flux in the parallelepiped is 0.563 times the threshold of the average in the slab, or the threshold of the maximum value of the flux in the parallelepiped is 1.39 times the threshold of the maximum value in the slab.

An interesting result from Eq. (4) is that the threshold for oscillation in the X-direction is not a function of the dimensions $Y$ and $Z$. (The parallelepiped threshold in the case of infinite $Y$ and $Z$ continues to differ from the slab because of the different boundary conditions at infinity.)

Reference *2* discussed a one-dimensional slab and the ordinate in Fig. 1 of ref. *2* is $\bar{\phi}_1 = \phi_{\text{av}}(\text{slab})$. The figure can also be applied in approximate calculations on parallelepipeds by using Eqs. (3) and (4).

The results can be generalized within the limitations of the linearized theory to a slab having nonuniform composition. The oscillation threshold of a slab in which $\phi$ is an arbitrary function $f(x)$ can be compared to the threshold in the corresponding parallelepiped in which

$$\phi = f(x) \sin (\pi y/Y) \sin (\pi z/Z).$$

Proceeding as before, but removing the integrals in $x$ which contain $f(x)$, we obtain

$$\phi_{max}(\text{par.}) = 1.39\ \phi_{max}(\text{slab}),$$

which is consistent with Eqs. (3) and (4) which apply only in the special case of uniform composition. The relations containing $\bar{\phi}_1$ and the average values cannot be obtained without specifying $f(x)$.

Similar results can be obtained in cylindrical geometry. The threshold values of $\bar{\phi}_1$ are given below for two modes of a bare uniform cylinder in which the steady state flux is

$$\phi = \phi_{max}(\text{cyl}) J_0\left(\frac{2.4048r}{R}\right) \sin\left(\frac{\pi z}{Z}\right).$$

For the first axial mode

$$g_1 = \frac{2.724}{R} J_0\left(\frac{2.4048r}{R}\right) \cdot \sqrt{2/Z} \sin\left(\frac{2\pi z}{Z}\right).$$

Substituting in Eq. (2) as before, the result is

$$\bar{\phi}_1 = 0.491\ \phi_{max}.$$

For the first azimuthal mode

$$g_1 = \frac{3.511}{R} J_1\left(\frac{3.832r}{R}\right) \cdot \sqrt{1/\pi} \sin\theta \cdot \sqrt{2/Z} \sin\left(\frac{\pi z}{Z}\right).$$

where $\theta$ is the azimuthal angle. Substituting in Eq. (2) the result is

$$\bar{\phi}_1 = 0.588\ \phi_{max}.$$

REFERENCES

1. A. G. Ward, CRRP-657 (1956).
2. D. Randall and D. S. St. John, *Nucleonics* **16** (3), 82–86, 129 (1958).

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## A Sulfur–Phosphorus Absolute Fast Neutron Flux Detector

Sulfur threshold detectors incorporating phosphorus for counting efficiency determination have been used at our laboratory. Detectors made from a mixture of a known ratio of phosphorus and sulfur were exposed to a known thermal flux producing radioactive $P^{32}$ from the $P^{31}$ $(n,\gamma)P^{32}$ reaction. The detectors were then counted in a standard beta counter and the counting efficiencies determined.

Cadmium covers were used to prevent further $P^{32}$ production by thermal neutron capture during activation as $S^{32}(n, p)P^{32}$ threshold detectors. Since the counting efficiency for each detector was known, the $P^{32}$ activity and therefore the absolute fast neutron flux could be determined.

The detectors were allowed to decay for at least three days before counting to eliminate the $Si^{31}$ 2.6 hr activity resulting from the P(n, p)Si reaction and the $Na^{24}$ 15 hr activity resulting from the Al(n, $\alpha$)Na reaction in the aluminum capsules used.

Results reproducible within about 20% were obtained with a mixture of one part red phosphorus to ten parts sulfur flour, and also with phosphorus pentasulfide ($P_2S_5$). The corrections required due to epicadmium production of $P^{32}$ by neutron capture were approximately 3% in the first case and 10% in the second. These corrections were estimated by activating similarly shaped cadmium covered phosphorus detectors in the fast flux to be measured and assuming the same counting efficiency as in the case of the sulfur–phosphorus detector.

It would be desirable to use a lower ratio of phosphorus to sulfur to minimize thermal activation during the fast flux measurement. This must be balanced against the statistical counting error introduced due to the low activity of the thermal flux calibration run.

The 2200 meters/sec cross section of phosphorus was taken to be $0.20 \pm 0.01$ barn. This is slightly higher than the value given by Hughes and Schwartz (*1*) since better values are available for the comparison standards on which their value is based (*2*). The energy dependence of the $S^{32}(n, p)P^{32}$ cross section spectrum was taken from Hughes and Schwartz (*1*) but normalized to give 65 mb for the mean cross section for a fission spectrum (*3*).

The principle of using thermal activation to produce an internal calibration might be applied to other elements. Two possibilities are the $Cl^{35}(n, \alpha)P^{32}$ reaction with an effective threshold at approximately 4 Mev and the $Al^{27}(n, \alpha)Na^{24}$ reaction with an effective threshold at approximately 8 Mev.

REFERENCES

1. D. J. Hughes and R. B. Schwartz, "Neutron Cross Sections," BNL 325, 2nd ed., Brookhaven National Laboratory (1958).
2. John R. Stehn, Brookhaven National Laboratory, private communication.
3. B. H. Parker, "The Measurement of Fast Neutron Flux Using the $S^{32}(n, p)P^{32}$ Reaction," AERE-R3443 (1960).

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## Resonance Absorption in Materials with Grain Structure (Addendum)

We have been asked how the results of a previous paper on the resonance absorption in materials with grain structure (*1*) (quoted as I) have to be modified, when the absorber grains occupy a sizeable fraction of the total volume. While it should be recognized that the assumption of random distribution is less good in this case, a formal answer is easily obtained.

We use the same notation as in I. Let $V_c$ be the volume fraction of the moderator matrix surrounding the absorber grains, and $V_a$ the volume fraction of the grains, so that

$$V_c + V_a = 1. \qquad (A.1)$$

Formerly it was assumed that $V_a \ll 1$, so that $V_c \approx 1$. Now