Letters to the Editors

The Kinetic Behavior of the Coupled Regions of the UTR-10 Reactor

The core of the UTR-10 (modified Argonaut type) reactor consists of two water moderated fuel regions imbedded in graphite, separated by approximately 18 in. and reflected by 12 in. of graphite. Each enriched U^{235} fuel region is subcritical when considered separately but reactivity coupling between regions allows the system to be critical.

Control of the reactor is accomplished by means of a shim-safety rod adjacent to one fuel region and a regulating rod adjacent to the other fuel region. The arrangement of the core is such that each control rod primarily affects the reactivity of the adjacent region, and hence, different amounts of negative reactivity can be introduced into each region. As a result, the average neutron fluxes in the two regions may be different. This condition, known as "flux tilting," has been reported by Baldwin (1) to occur in the Argonaut reactor when the fuel is loaded in the two-slab configuration.

The effects of flux tilting upon the kinetic behavior of the UTR-10 reactor were studied using an analog computer. The two core regions were treated as separate "point" reactors with reactivity coupling. Step and ramp inputs of reactivity to one region as well as a sinusoidal variation of the coupling were simulated on the computer.

The reactor kinetics equations used to describe the time behavior of each fuel region of the UTR-10 reactor were:

$$\frac{1}{v}\frac{d\phi_1}{dt} = \frac{\delta k_1 \phi_1}{vl} - \frac{\beta\phi_1}{vl} + \frac{\alpha\phi_2(t-\tau)}{vl} + \lambda c_1 \tag{1}$$

$$\frac{1}{v}\frac{d\phi_2}{dt} = \frac{\delta k_2 \phi_2}{vl} - \frac{\beta\phi_2}{vl} + \frac{\alpha\phi_1(t-\tau)}{vl} + \lambda c_2 \tag{2}$$

$$\frac{dc_1}{dt} = \frac{\beta\phi_1}{vl} - \lambda c_1 \tag{3}$$

$$\frac{dc_2}{dt} = \frac{\beta\phi_2}{vl} - \lambda c_2 \tag{4}$$

where v is neutron velocity, τ is a time lag which is discussed later, and α is the coupling coefficient between the two regions denoted by the subscripts 1 and 2. The terms $\alpha\phi_1(t-\tau)/vl$ and $\alpha\phi_2(t-\tau)/vl$ account for the coupling interaction between regions. Because of the time required for a disturbance introduced into one region to affect the time behavior of the opposite fuel region, the coupling interaction in one region was assumed to be proportional to the average neutron flux in the opposite side at a previous time $(t-\tau)$. An approximate expression for $\phi(t-\tau)$ was obtained by expanding it in a series. The first two terms are

$$\phi(t-\tau) = \phi(t) - \left(\frac{d\phi}{dt}\right)\tau \tag{5}$$

To evaluate the lag time, τ, it was assumed that a flux disturbance originating in one of the fuel regions travels across to the opposite side with the velocity of a neutron wave. Work by Uhrig (2) indicated that the velocity of a neutron wave in graphite is approximately constant at 1.4×10^4 cm/sec for frequencies from 0.1 to 100 rad/sec. The lag time—the time required for a wave to traverse the 18 in. of graphite between the fuel regions—is estimated to be 0.0033 sec. For the conditions considered in this study, the effect of this lag time was neglected.

Equations (1) through (4) were evaluated and scaled in terms of computer voltages for solution on an analog computer using an effective neutron lifetime l of 135 μsec. Solutions for step and ramp inputs of reactivity and for sinusoidal variations of the coupling coefficient with and without flux tilting present in the reactor were obtained.

When a step input of reactivity is added to only one core, the "prompt jump" in neutron flux is greater in that core, resulting in flux tilting. However, the stable periods of the two cores are equal. When flux tilting is present initially, the addition of a step input of reactivity into the high flux region results in a greater average rate of flux increase and a shorter stable period than when the same step input of reactivity is added to the low flux region.

The response of the reactor to a ramp input of reactivity into one of the fuel regions was investigated. This study was carried out with the flux tilted so that the reactivity ramp was added to the high flux region, to the low flux region, and without flux tilting. The results indicate that the effect of the reactivity ramp input upon the neutron density depends upon the degree of flux tilting. For the same total amount of reactivity (representing the shim-safety rod) added at the same rate, (% δk/sec) the flux level at the end of the ramp is higher by a factor of approximately five than when the ramp input is into the higher flux region for $\phi_1/\phi_2 = 2$. These results serve to point out that the "worth" of a control rod used in the two region system varies with the degree of flux tilting across the reactor.

A preliminary estimate of the degree of flux tilting that can be developed was made using the equivalent inhour curves for the two region systems. Figure 1 shows the relationship between δk_1, and δk_2 for critical conditions and for a stable reactor period of 50 sec. These curves were obtained for a value of 0.0155 (supplied by the manufacturer[1]) for the coupling coefficient of the UTR-10 re-

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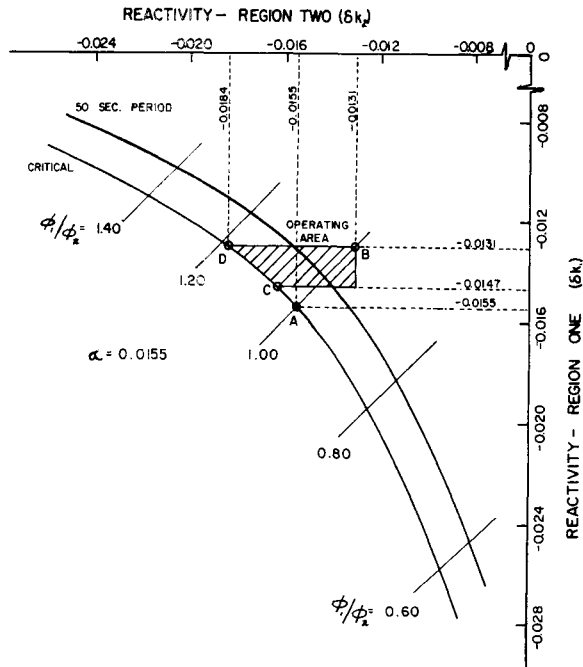
FIG. 1. Equivalent inhour curves for a two region reactor core.

actor. For critical conditions and for a neutron flux ratio of unity

$$\delta k_1 = \delta k_2 = -0.0155 \qquad (6)$$

Each region is subcritical by this amount, but coupling between regions allows the system to be critical. This condition is represented by point A in Fig. 1. The excess reactivity of the system is 0.48%, and it is assumed that this excess reactivity is divided equally between the fuel regions. The maximum values of reactivity for equal fluxes in the two regions are

$$\delta k_1 = \delta k_2 = -0.0155 + 0.0024 = 0.0131 \qquad (7)$$

which is represented by point B in Fig. 1. The two extreme critical operating positions for the control rods occur when the regulating rod is completely inserted or completely withdrawn. If it is assumed that a control rod only affects the reactivity of the adjacent fuel region and the regulating rod in region one which is worth 0.16% is completely inserted, then

$$\delta k_1 = -0.0131 - 0.0016 = -0.0147 \qquad (8)$$

The reactor would then be maintained in a critical condition by adjusting the shim-safety rod adjacent to region two. The reactor is operating at the condition represented by point C of Fig. 1, and the ratio ϕ_1/ϕ_2 is observed to be 1.05. The other extreme operating point is found when the regulating rod is completely withdrawn from the core, and

$$\delta k_1 = -0.0131 \qquad (9)$$

Critical operation is now represented by the point D of Fig. 1 with a ratio of ϕ_1/ϕ_2 of 1.18.

The ratio of ϕ_1/ϕ_2 varies from 1.05 to 1.18 for critical conditions with the higher flux in the region adjacent to

the regulating rod. The shaded area of Fig. 1 represents the range over which δk_1, and δk_2 can be varied by movement of the regulating and shim rods when the UTR-10 reactor is critical or supercritical.

The response of the reactor to a sinusoidal variation of coupling having the form

$$\alpha = \alpha_0 + \delta\alpha \sin \omega t \qquad (10)$$

was investigated. An oscillator consisting of a stator and rotor with cadmium patterns placed in the graphite between the fuel regions would produce a variation of coupling of approximately this form. It was assumed that the oscillator would affect only the coupling between regions and not the individual reactivity of each region.

The reactor transfer functions without flux tilting present are identical for each region, and for one group of delayed neutrons can be shown to be approximately

$$\frac{\delta\phi_1(s)}{\delta\alpha(s)} = \frac{\delta\phi_2(s)}{\delta\alpha(s)} = \frac{\phi_{10}}{l}\left[\frac{(s+\lambda)}{s[s+(\beta/l)]}\right] = \frac{\phi_{20}}{l}\left[\frac{(s+\lambda)}{s[s+(\beta/l)]}\right] \qquad (11)$$

It is essentially the same as the transfer function for a single region reactor with a sinusoidal input of δk. Break frequencies occur at λ and β/l.

When flux tilting is present the transfer functions for regions 1 and 2 can be shown to be

$$\frac{\delta\phi_1(s)}{\delta\alpha(s)} = $$

$$\phi_{10} \frac{\left|\begin{bmatrix}\phi_{20}\\\phi_{10}\end{bmatrix} \quad \begin{bmatrix}-\dfrac{\alpha_0}{l}\end{bmatrix}\\[2mm] [1] \quad \dfrac{\left[\left(s+\dfrac{\beta-\delta k_2}{l}\right)\left(s-\dfrac{\lambda\delta k_2}{\beta-\delta k_2}\right)\right]}{(s+\lambda)}\right|}{|\Delta|} \qquad (12)$$

$$\frac{\delta\phi_2(s)}{\delta\alpha(s)} = $$

$$\phi_{20} \frac{\left|\dfrac{\left[\left(s+\dfrac{\beta-\delta k_1}{l}\right)\left(s-\dfrac{\lambda\delta k_1}{\beta-\delta k_1}\right)\right]}{(s+\lambda)} \quad [1]\\[2mm] \begin{bmatrix}-\dfrac{\alpha_0}{l}\end{bmatrix} \quad \begin{bmatrix}\phi_{10}\\\phi_{20}\end{bmatrix}\right|}{|\Delta|} \qquad (13)$$

where

$$|\Delta| = $$

$$\left|\dfrac{\left[\left(s+\dfrac{\beta-\delta k_1}{l}\right)\left(s-\dfrac{\lambda\delta k_1}{\beta-\delta k_1}\right)\right]}{(s+\lambda)} \quad \begin{bmatrix}-\dfrac{\alpha_0}{l}\end{bmatrix}\\[3mm] \begin{bmatrix}-\dfrac{\alpha_0}{l}\end{bmatrix} \quad \dfrac{\left[\left(s+\dfrac{\beta-\delta k_1}{l}\right)\left(s-\dfrac{\lambda\delta k_1}{\beta-\delta k_1}\right)\right]}{(s+\lambda)}\right| \qquad (14)$$

It should be noted that both δk_1 and δk_2 are negative quan-

tities which can be evaluated by

$$\delta k_1 = -\alpha_0 \left(\frac{\phi_2}{\phi_1} \right) \qquad (15)$$

$$\delta k_2 = -\alpha_0 \left(\frac{\phi_1}{\phi_2} \right) \qquad (16)$$

The computer values for gain and phase lag of the reactor transfer function with flux tilting $\phi_1/\phi_2 = 1.2$ present, which represents the most severe flux tilting possible in the UTR-10 reactor, were not significantly different from those for a reactor without flux tilting. Additional break frequencies are introduced into the reactor transfer function by the flux tilting, but they occur at about the same frequency for the terms in the numerator and denominator. Consequently, no major change in the gain or phase shift results. The significant break frequencies still occur at λ and β/l.

The significant conclusions to be drawn from this study are:

1. Flux tilting is possible in the UTR-10 reactor. However, both fuel regions exhibit the same stable reactor period.

2. Flux tilting can affect the worth of a control rod but, due to the limited range of tilting that is possible in the UTR-10 reactor, variations of rod worth due to flux tilting should not be significant. The maximum degree of flux tilting occurs when the regulating rod is completely withdrawn.

3. A flux ratio of unity is never attained in the UTR-10 reactor for an excess reactivity of 0.48%, when the reactor is "just critical." The ratio of ϕ_1 to ϕ_2 varies from 1.05 to 1.18 with the higher flux in the region adjacent to the regulating rod.

4. The open loop response of the UTR-10 reactor to a sinusoidal variation of coupling, with or without flux tilting present, is not significantly different than the response of a single region reactor to a sinusoidal variation of reactivity.

REFERENCES

1. GEORGE C. BALDWIN, *Nuc. Sci. Eng.* **b**, 320–327 (1959).
2. ROBERT E. UHRIG, *Trans. Am. Nucl. Soc. Vol.* **2**, No. 2 Paper 7-2 (1959).

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 Received August 6, 1960
 Revised January 15, 1963

The Convergence of the Equipoise Method

The difference equations corresponding to the group-diffusion method may be written in the form

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$$L\varphi = \lambda M\varphi \qquad (1)$$

Here φ is a flux-vector whose dimension N is equal to the number of groups multiplied by the number of interior points in the reactor. L and M are $N \times N$ matrices. L has nonpositive off-diagonal elements, while the diagonal is positive. Due to the facts that the absorption cross sections are nonnegative, and that one may describe any multiplicative transfer of neutrons from one group to another by means of the fission matrix M, the diagonal of L dominates vertically (with strict dominance for at least one column). M is a nonnegative matrix. For a more detailed description of these matrices, see for instance ref. *1*.

One may construct a sequence of splittings

$$L = A_j + (L - A_j), \qquad (j = 1, 2, \cdots) \quad (2)$$

with nonsingular matrices A_j, and define an iterative procedure

$$A_j\varphi_j = (A_j - L + \lambda_{j-1}M)\varphi_{j-1} \qquad (j = 1, 2, \cdots) \quad (3)$$

$$\lambda_j = f(\varphi_j) \qquad (4)$$

for determining the numerically smallest eigenvalue λ and the corresponding positive eigenvector φ. Here it must be required that the function f in (4) is chosen so that the substitution $\varphi_j = \varphi$ yields $\lambda_j = \lambda$.

Now it is well known that the scheme (3) converges independently of the f-function chosen, if $A_j = L$, corresponding to one of the usually employed methods, the power method. For two- or three-dimensional calculations the matrix L is not easily solvable, and (3) is solved iteratively by means of the so-called inner iterations. In most practical cases (at least for a two-group model) the eigenvalues of the matrix $L^{-1}M$ lie in the vicinity of the positive part of the real axis (although examples of reactors with negative or complex eigenvalues may be found) (*2*). Then one may use an iterative scheme, a little more elaborate than (3), the so-called Chebyshev-method (see for instance *3*).

The versions of the Equipoise method known to the author of this letter (*4, 5*) all utilize a basic scheme of the form (3), (4) with $A_j \neq L$. Such a scheme may diverge (see the examples given below). But usually A_j is chosen so that the matrix $A_j^{-1} (A_j - L)$ has a small spectral radius. In this case (3) could be regarded as an effective first inner iteration to solve (1) in the usual way as described above, and one has a fair hope of success.

However, for $A_j \neq L$ the convergence of the scheme (3), (4) may depend on the function f. The choice made in the Equipoise codes,

$$\lambda_j = \frac{e^T L\varphi_j}{e^T M\varphi_j}, \qquad (5)$$

where $e^T = (1; 1; \cdots ; 1)$, seems to be a good one. It ensures that λ_j is positive, if $\varphi_j > 0$, without using numerical values of possibly negative numbers, as some other methods would require. This is a definite advantage, as will be clear from the following.

It is now assumed (for simplicity) that $A_j^{-1} \geq 0$ and $A_j - L + \lambda_{j-1} M \geq 0$ for all j. The initial vector φ_0 is chosen positive, which ensures that all $\varphi_j \geq 0$. If the scheme (3)–(5) converges, it must converge to the unique positive eigenvector φ. If it should diverge, there are various possibilities.