Letters to the Editors

Tables of Secant Integrals of the First and Second Kinds

A class of integral functions which one frequently must evaluate in working shielding problems and other problems involving the attenuation of radiation through shields is the class known as the Secant Integrals.

This class of functions can be mathematically defined as

$$I_{n+1}(\theta, b) = b^n \int_0^\theta (\sec \theta')^n e^{-b \sec \theta'} \, d\theta', \quad n = 0, 1, 2, 3, \cdots \quad (1)$$

where

$b = \mu t$, single material
$b = \sum_i \mu_i t_i$, $i = 1, 2, \cdots$ for a number of layers
μ_i = mass absorption coefficient for the ith material
t_i = material thickness of the ith material.

$$I_1(\theta, b) \equiv F(\theta, b) = \int_0^\theta e^{-b \sec \theta'} \, d\theta. \quad (2)$$

Curves of these integrals have been published in Rockwell, "Shielding Design Manual," TID-7004 (1956).

$$I_2(\theta, b) \equiv G(\theta, b) = b \int_0^\theta \sec \theta' e^{-b \sec \theta'} \, d\theta'. \quad (3)$$

This class of integrals arises wherever a buildup function $B(b, \theta)$, which appears beneath the integral sign, is approximated by a polynominal expression. For example, if we have a function $\Gamma(\theta, b)$ such that

$$\Gamma(\theta, b) = \Gamma_0 \int_0^\theta B(b, \theta) e^{-b \sec \theta} \, d\theta \quad (4)$$

where $B(b, \theta)$ is approximated as

$$B(b, \theta) = 1 + \alpha b \sec \theta. \quad (5)$$

$$\alpha = \text{constant.}$$

From the definition of the $F(\theta, b)$ and $G(\theta, b)$ functions, Eqs. (2) and (3), Eq. (6) may be rewritten

$$\Gamma(\theta, b) = \Gamma_0[F(\theta, b) + \alpha G(\theta, b)]. \quad (6)$$

$$I_3(\theta, b) \equiv H(\theta, b) = b^2 \int_0^\theta \sec^2 \theta' e^{-b \sec \theta'} \, d\theta'. \quad (7)$$

These integrals arise when the expression for buildup is written

$$B(\theta, b) = 1 + \alpha b \sec \theta + \beta b^2 \sec^2 \theta; \alpha, \beta = \text{constant.} \quad (8)$$

Then the function $\Gamma(\theta, b)$ in Eq. (4) may be written

$$\Gamma(\theta, b) = \Gamma_0[F(\theta, b) + \alpha G(\theta, b) + \beta H(\theta, b)]. \quad (9)$$

The approximation given in Eq. (5) is sufficient for most shielding calculations which are used for practical application. The secant integrals of the first and second kinds were computed by numerical integration of Eqs. (2) and (3) on an IBM-650 computer, and have been used extensively over the past several years in shielding calculations at Atomic Power Development Associates, Inc. They are available from that organization in tabular form.

I wish to acknowledge with grateful appreciation the work of APDA's Computer Group who programmed the integrals, checked the results, and proofed the tables. Especial thanks are due Miss Yvonne Wilson and former employee Miss Agnes Leidel for their considerable efforts on this project.

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Effective Resonance Integral Dependence on the Moderator Slowing Down Properties

The effective resonance integral calculated according to the usual approximations NR (narrow resonance) or NRIA (narrow resonance infinite mass absorber) is independent of the moderator slowing down properties (e.g., *1*). As the calculation of the heterogeneous assembly is usually reduced to the calculation of the modified homogeneous mixture, the above statement holds in both cases. It also seems that beyond the experimental errors no influence of the moderator on the measured resonance integrals is found.

In this paper we wish to show by exact calculation of the resonance absorption in an infinite homogeneous mixture for resolved resonances of U^{238} how far the moderator slowing down properties and the interference scattering actually influence the effective resonance integral and its temperature coefficient. We will also compare the exact results with the usual NR and NRIA approximations.

The exact resonance absorption is calculated by numerical solution of the neutron slowing down equation for a mixture of elements:

$$F(u) = \sum_n \frac{1}{1 - \alpha_n} \int_{u-\epsilon_n}^{u} \frac{\Sigma_{sn}(u')}{\Sigma(u')} F(u') e^{u'-u} \, du'. \quad (1)$$

The symbols have the following meaning: $F(u) = \phi(u)\,\Sigma(u)$, the collision rate density; $\phi(u)$ is the flux of neutrons per unit lethargy at lethargy u; $\Sigma(u) = \sum_n [\Sigma_{sn}(u) + \Sigma_{an}(u)]$ is the total macroscopic cross section at lethargy u; $\Sigma_{sn}(u)$ is the macroscopic scattering cross section of the nth element including its potential scattering; $\Sigma_{an}(u)$ is the macroscopic absorption cross section of the nth element; $\alpha_n = [(A_n - 1)/(A_n + 1)]^2$, A_n is the atomic mass number of the nth element; $\epsilon_n = \ln 1/\alpha_n$. The suitable form for