TABLE IV

ANALYTICAL REPORT ON Pu^{239} SAMPLE[a]

Al[b]	9500	Gd < 6	Dy < 24
C	85	B < 0.5	I < 60
O	55	Sm < 60	Hg < 80
Cr	30	Eu < 12	In < 60
Fe	275	Cd < 48	Rh < 60
Ni	20	Li < 2	Er < 24
	Total Pu 98.90 ± .2%		

[a] All values reported in parts per million by weight.

[b] The sample was an alloy containing 1% by weight aluminum.

TABLE V

ISOTOPIC ANALYSIS OF Pu^{239} SAMPLE

Pu^{238}	0.003%
Pu^{239}	97.90%
Pu^{240}	2.01%
Pu^{241}	0.06%

$\pm \frac{0.006}{0.009}$. Similarly, the ratio for Pu^{239} and U^{233} is calculable but redundant.

REFERENCES

1. R. L. MACKLIN, G. DESAUSSURE, J. D. KINGTON, AND W. S. LYON, *Nuclear Sci. and Eng.* **8**, 210–220 (1960).
2. R. L. MACKLIN, G. DESAUSSURE, J. D. KINGTON, AND W. S. LYON, Oak Ridge National Laboratory Report, ORNL-CF-60-2-84 (1960).
3. G. DESAUSSURE, R. L. MACKLIN, J. D. KINGTON AND W. S. LYON, 1959 Annual Meeting of the ANS, Gatlinburg, Tennessee, June 15-17 (1959). Session No. 18, Paper 2A. This paper did not appear in the Transactions of the meeting because it was a post-deadline paper.
4. M. A. LEVINE, *Nuclear Sci. and Eng.* **7**, 545 (1960).
5. R. RAMANNA, N. SARMA, C. S. SOMANATHAN, K. USHA AND G. VENKATARAMAN, *Proc. Second Intern. Conf. Peaceful Uses Atomic Energy, Geneva*, 15/P/1636 (1958).

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Effect of Zero Gradient Boundary Conditions on Cell Calculations in Cylindrical Geometry*

In a recent letter, Thie (*1*) has pointed out the disagreement between computations by various methods of the disadvantage factors of cylindrical rods in certain square lattices. The lattices considered are tightly packed and have low density moderators. The methods which appear to give the most accurate values are a Monte Carlo treatment and Amouyal's method (*2*) which are in approximate agreement. One of the methods with results in disagreement with these values is a P_3 approximation which uses reflecting boundary conditions on a cylindrical surface such that the area of the cylindrical cell is identical with that of the square cell of the physical lattice. An analogous P_1 approximation may also be carried out for the cylindrical cell. Disadvantage factors by these four methods for four of Thie's lattices are shown in Table I. The P_3 values represent an independent check on Thie's calculation in agreement with his results. The disagreement between the P_3 value for reflecting boundary conditions and the Monte Carlo value is small for Lattice 1, but substantial for the smaller radius of Lattice 4, and has the anomalous feature that it differs in the opposite sense from that of the P_1 approximation and by a larger amount. This type of discrepancy has been noted by Newmarch (*3*) who has shown that the disadvantage factor for such a P_3 approximation does not approach unity as the cell dimension becomes small. Similar considerations have also been given by Daitch *et al.* (*5*).

TABLE I

DISADVANTAGE FACTORS BY VARIOUS METHODS

Lattice	Monte Carlo	Amouyal	P_1	P_3 (Reflecting boundary conditions)
1	1.135	1.170	1.051	1.166
2		1.169	1.039	1.189
3		1.155	1.036	1.207
4	1.137	1.159	1.030	1.265

Several points should be made about the reflecting boundary conditions applied on the cylindrical surface. As Thie indicates, these conditions are an artificiality which cannot be realized physically in the case of neutrons. The physical surfaces for which reflecting conditions are implied by the geometry are the flat surfaces of the square cell.

In contrast to the case of plane geometry, reflecting boundary conditions on the cylindrical surface do not imply that the derivative of the scalar flux vanishes, except in the special case of the P_1 approximation. For the lattice in question reflecting conditions imply a large and unrealistic value of this derivative. Figure 1 shows a flux plot across the cylindrical cell for Lattice 4. As the homogeneous case is approached the physical flux must become flat across the cell, and it is clear that a flux shape like that of Fig. 1 cannot be applicable.

In the notation of (*4*), the reflecting boundary conditions for a P_3 approximation have the form

$$\psi_{1,1} = \psi_{3,1} = \psi_{3,3} = 0. \tag{1}$$

The component $\psi_{1,1}$ is the neutron current and the condition $\psi_{1,1} = 0$ must be retained in any cell calculation. The second condition suggested by the above considerations is $d\psi_{0,0}/dr = 0$, $\psi_{0,0}$ being the scalar flux.[1] A third condition may be obtained in several ways; the results have proved insensitive

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[1] Cell boundary conditions using these two conditions have been used previously (compare ref. *5*).

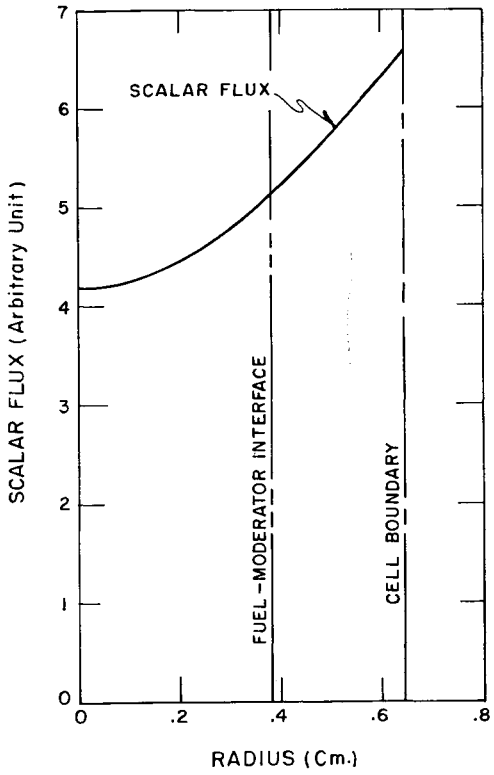

FIG. 1. P_3 scalar flux corresponding to reflecting conditions at the boundary of the cylindrical cell of Lattice 4.

TABLE II

P_3 DISADVANTAGE FACTORS FOR $d\psi_{0,0}/dr = 0$

Lattice	$\psi_{1,1} = 0$, $d\psi_{0,0}/dr = 0$		
	$\psi_{3,3} = 0$	$\psi_{3,1} = 0$	$\psi_{3,1} + 0.1\,\psi_{3,3} = 0$
1	1.103	1.099	1.100
2	1.074	1.077	1.077
3	1.086	1.075	1.075
4	1.050	1.059	1.059

to the particular choice for the sets of conditions used. Disadvantage factors for the four lattices and for three forms of the third condition are given in Table II. Each of the sets of boundary conditions is such as to reduce to reflecting boundary conditions as the radius of the cylindrical surface becomes large.

It is evident that the disadvantage factor is substantially the same for the three forms of the third condition. The results for the conditions of Table II agree more closely with the Monte Carlo value for Lattice 4 than does the result given in Table I for reflecting boundary conditions. For Lattice 1, the discrepancies are comparable. However, the most significant fact is that the values corresponding to $d\psi_{0,0}/dr = 0$ are intermediate between the disadvantage factors of the P_1 approximation and those of the Monte Carlo method and Amouyal's method.

The dependence of the calculated disadvantage factor on the type of cell boundary conditions used is emphasized by the values obtained. It would be incorrect to conclude from results based on reflecting boundary conditions that a P_3 or higher approximation is necessarily less accurate than a P_1 approximation. On the contrary, it appears that the anomalous inaccuracies found in the P_3 approximation are due to the use of reflecting boundary conditions rather than conditions which incorporate $d\psi_{0,0}/dr = 0$.

REFERENCES

1. J. A. THIE, *Nuclear Sci. and Eng.* **9,** 286 (1961).
2. A. AMOUYAL, P. BENOIST, ET J. HOROWITZ, *J. Nuclear Energy* **6,** 79 (1957).
3. D. A. NEWMARCH, AEEW-R 34 (1960).
4. J. M. DODERLEIN AND K. BRYHN-INGEBRIGTSEN, *Proc. 2nd Conf. Peaceful Uses Atomic Energy, Geneva,* P/570 (1958).
5. P. B. DAITCH, B. J. MCGOVERN, AND M. SCHILDER, Combustion engineering MPC-20, TID 6940, p. 11 (1959).

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