large values of $n$. It can, therefore, be concluded that the eigenvalues approach integers; i.e., $\alpha_n{}^2 \to n$ for $n \to \infty$.

REFERENCES

1. H. Hurwitz, Jr. and M. S. Nelkin, *Nuclear Sci. and Eng.* **3,** 1 (1958).
2. M. S. Nelkin, *J. Nuclear Energy* **8,** 48 (1958).
3. P. Michael, *Nuclear Sci. and Eng.* **8,** 426 (1960).
4. N. Corngold and L. Zamick, *Trans. Am. Nuclear Soc.* **3,** 504 (1960).
5. W. Häfele and L. Dresner, *Nuclear Sci. and Eng.* **7,** 304 (1959).
6. L. De Sobrino and M. Clark, Jr., *Nuclear Sci. and Eng.* **10,** 388 (1961).
7. H. Bateman, "Tables of Integral Transforms," Vol. II, p. 292. Compiled by the Staff of the Bateman Project, A. Erdelyi, Editor. McGraw Hill, New York, 1953.
8. P. M. Morse and H. Feshbach, "Methods of Theoretical Physics," Part II. McGraw Hill, New York, 1953.

E. Garelis

*Vallecitos Atomic Laboratory*
*General Electric Company*
*Received November 28, 1961*
*Revised January 31, 1962*

## Effective Diffusion Coefficient in Void Regions

The multigroup diffusion theory for a virtually critical medium, with homogeneous regions, gives rise to the following system of differential equations (*1, 2*).

$$D_i^l \nabla^2 \phi_i^l - \left(\Sigma_{ia}^l + \sum_{j=i+1}^{g} \Sigma_{i\to j}^l\right)\phi_i^l + \sum_{j=1}^{i-1} \Sigma_{j\to i}^l \phi_i^l + \frac{1}{K}\sum_{j=1}^{g} f_i^l (\nu\Sigma_j)_j^l \phi_j^l = 0 \qquad (1)$$

$i = 1, 2, \cdots, g$ (number of groups); $l = 1, 2, \cdots r$ (number of regions), with the boundary conditions of continuity of fluxes and currents at the interfaces.

This system, when applied in one dimensional cylindrical geometry to a void region, $l = v$, with $\Sigma_{xi}^v = 0$, $x = a$, $i \to j, f$, and $D_i^v \to \infty$ (it is assumed to be arbitrarily large in the codes WANDA, AIM-5, $\cdots$), results in

$$D_i^v \nabla^2 \phi_i^v = D_i^v \frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi_i^v}{dr}\right) = 0 \qquad (2)$$

then

$$\phi_i^v(r) = a_i^v \ln r + b_i^v \qquad (3)$$

with the boundary conditions, at the inner interface of radius $r_{v-1} \neq 0, \infty$

$$D_i^{v-1} \nabla\phi_i^{v-1}(r_{v-1}) = D_i^v \nabla\phi_i^v(r_{v-1}) = D_i^v a_i^v r_{v-1}^{-1} \qquad (4)$$

$$\phi_i^{v-1}(r_{v-1}) = \phi_i^v(r_{v-1}) = a_i^v \ln r_{v-1} + b_i^v \qquad (5)$$

This system of equations determines $a_i^v$, $b_i^v$. When $D_i^v \to \infty$, $a_i^v \to 0$, a flux $\phi_i^v \to b_i^v$ = constant is obtained.

Depending on whether the net current flow through the gap is inwards or outwards, this theory overestimates or underestimates the fraction of neutrons entering the void from the outer interface which reaches the inner interface. Actually, current flow through the gap produces a discrete jump in the value of the flux which has been calculated by Newmarch (*3*) to be

$$\phi_i^v(r_v) - \phi_i^v(r_{v-1}) = 2\alpha D_i^{v-1}\nabla\phi_i^{v-1}(r_{v-1}) \qquad (6)$$

with

$$r_v D_i^v \nabla\phi_i^v(r_v) - r_{v-1} D_i^v \nabla\phi_i^v(r_{v-1}) = 0 \qquad (7)$$

and

$$\alpha = 1 - \frac{2}{\pi}\arcsin\frac{r_{v-1}}{r_v} - \frac{2}{\pi}\frac{r_{v-1}}{r_v}\left(1 - \frac{r_{v-1}^2}{r_v^2}\right)^{1/2} \qquad (8)$$

For this correction to be applied more easily, an effective diffusion coefficient in the void can be considered, which must satisfy Eqs. (4) and (6). Equation (5) can be used to determine $b_i^v$, and Eq. (7) is always satisfied along with Eq. (3).

Substitution of Eq. (3) into (6), gives

$$a_i^v = \frac{2\alpha D_i^{v-1}\nabla\phi_i^{v-1}(r_{v-1})}{\ln(r_v/r_{v-1})} \qquad (9)$$

which substituted into Eq. (4), gives in turn

$$D_{\text{eff}}^v = \frac{r_{v-1}\ln(r_v/r_{v-1})}{2\alpha} = r_{v-1} f(r_{v-1}/r_v) \qquad (10)$$

with

$$D_{\text{eff}}^v \to \infty, \quad \text{if } r_{v-1} \to r_v; \quad D_{\text{eff}}^v = (1 \pm 0.2) r_{v-1}, \quad \text{if } 0,15 \leqq r_{v-1}/r_v \leqq 0,85 \qquad (11)$$

Therefore, in one-dimensional cylindrical multigroup diffusion equations, void may be represented by purely diffusive media, with cross sections equal to zero, and effective diffusion coefficient given by Eq. (10). In this way the Newmarch correction is taken into account.

REFERENCES

1. O. J. Marlowe, WAPD-TM-28 (1956).
2. H. P. Flatt, D. C. Baller, and E. R. Cohen, NAA-SR-4694 (1960).
3. D. A. Newmarch, *J. Nuclear Energy* **2,** 52–58 (1955).

G. Velarde

*Div. Reactores*
*Junta de Energía Nuclear*
*Madrid, 3, Spain*
*Received November 28, 1961*