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Letters to the Editors

Reactor Size Sufficient for Stability Against Spatial Xenon Oscillations*

Several papers (*1–3*) have indicated the existence of a critical reactor size which is absolutely stable independently of flux level against higher spatial mode xenon oscillations in the presence of zero or negative temperature feedback. The desired criterion (Eq. (5)) will be shown to follow from the exact solution of the usual one neutron energy group model. Ward (*3*) stated a slightly weaker result which followed from the linearized theory in the limit of infinite flux. Ward's result also follows from the general theory of multiplying systems (*4*) if we define a criticality factor for each spatial mode as:

$$k_n \equiv \frac{k_\infty}{1 + M_0^2 B_n^2} \qquad n = 0, 1, \cdots , \qquad (1)$$

A sufficient condition for stability of the nth mode is $k_n < 1$. If then $k_0 = 1 + \delta_0$ then $k_\infty = (1 + M_0^2 B_0^2)(1 + \delta_0)$ and the stability condition for higher modes is

$$\frac{M_0^2[B_n^2 - B_0^2]}{1 + M_0^2 B_0^2} > \delta_0 . \qquad (2)$$

Physically this states that the leakage in the nth mode must be greater than some critical value in order for the nth mode to be stable.

In the absence of temperature effects and at very high flux the general kinetic equations in parametric form are

$$\begin{aligned} \tau_{e0} \frac{\partial \Phi}{\partial t} &= M_0^2[\nabla^2 \Phi + B_0^2 \Phi] + \frac{y}{c}\, [1 - X]\Phi \\ \frac{\partial I}{\partial t} &= \lambda_i(\Phi - I) \\ X\Phi &= Y_x \Phi + Y_I\, I \end{aligned} \qquad (3a)$$

where y/c is the equilibrium xenon poisoning at very high flux, τ_{e0} is the effective neutron lifetime including delayed neutrons, Y_X and Y_I are the relative prompt yields of xenon and iodine with $Y_X + Y_I = 1$, B_0^2 is the geometric buckling, and M_0^2 is the migration area. In this limit of high flux, the set (3a) is linear in Φ and $X\Phi$ and may be solved exactly. Subtracting out the equilibrium solution, and applying a Laplace transform, Eq. (3a) reduces to

$$\begin{aligned} \nabla^2 \tilde{\psi} + [B_0^2 + A(s)]\tilde{\psi} &= 0 \\ A(s) &= \frac{1}{M_0^2}\left[\frac{y}{c}\left(1 - Y_x - \frac{Y_I \lambda_i}{s + \lambda_i}\right) - \tau_{e0}\, s\right]. \end{aligned} \qquad (3b)$$

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Since (3b) is linear-homogeneous, with constant coefficients, $\nabla^2 \tilde{\psi} = -B^2 \tilde{\psi}$ for all separable coordinate systems. For any spatial mode, the condition for solution of (3b) is

$$[-B_n^2 + B_0^2 + A(s)] = 0. \qquad (4)$$

Hence, the condition for stability of the nth mode is then

$$M_0^2[B_n^2 - B_0^2] > \frac{y}{c}\,(1 - Y_x) - \lambda_i\, \tau_{e0} . \qquad (5)$$

In Eq. (2), the definition of δ_0 for this model could have led us to the first term on the right hand side equally well if the effect of prompt xenon was considered simply subtractive. The second term may be accounted for on the grounds that even delayed xenon would appear to be prompt and therefore stabilizing if the iodine decay time was short compared to the reactor period. However, the effect is small in practical cases. The value of c used to convert local xenon absorptions per fission to reactivity loss is taken as

$$c \equiv \frac{1 + \alpha}{f} .$$

TABLE I

Exact Stability Criteria of the First Harmonic for Various Fuels in Bare and Zero Leakage Reactors

Fuel	Axial		Diametral	
	Bare $B_1^2 = (2\pi/h)^2$	Zero leakage $B_1^2 = (\pi/h)^2$	Bare $B_1^2 = \left(\frac{3.832}{r_0}\right)^2$	Zero leakage $B_1^2 = \left(\frac{1.842}{r_0}\right)^2$
Natural U	h/M_0 <31.5	<18.2	d/M_0 <34.4	<21.3
3% U^{235}	26.6	15.4	29.1	18.0
U^{235}	25.0	14.4	27.3	16.9
U^{233}	26.4	15.3	28.8	17.8
Pu^{239}	27.1	15.7	29.6	18.3

Stability conditions for various fuels are listed in Table I for the first axial and diametral harmonics of bare and zero leakage reactors. The values of Y_X, Y_I, c, and τ_{e0} are taken from Table I of (*5*).

The author acknowledges valuable discussion with Dr. Paul Michael who suggested the possibility of a relation between the generalized multiplication constant and the stability criteria of the linear theory.

REFERENCES

1. A. G. WARD, The problem of flux instability in large power reactors. CRRP-657 (1956).
2. D. RANDALL AND D. S. ST. JOHN, Xenon spatial oscillations. *Nucleonics* **16,** 82 (1958).
3. R. M. PEARCE, Analog simulation of xenon instability. CRRP-998. (1961)
4. S. GLASSTONE AND M. C. EDLUND, "The Elements of Nuclear Reactor Theory." Van Nostrand, Princeton, N. J., 1952.
5. J. CHERNICK, G. S. LELLOUCHE, AND W. WOLLMAN, The effect of temperature on xenon instability. *Nuclear Sci. and Eng.* **10,** 120 (1961).

GERALD S. LELLOUCHE

Brookhaven National Laboratory,
Upton, N. Y.

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Application of the Pile Oscillator to Large-Neutron-Dose Measurement

Determination of large neutron dose is indispensable to the estimate of the burnup of nuclear fuel and of the yield of radioactive isotopes in the reactor, and to other evaluations in experiments related to irradiation by a large quantity of neutrons.

The mass-spectrometric method (*1*) is one of the standard techniques of measuring a high thermal-neutron absorption cross section. It is also applicable to the large-neutron-dose measurement: a piece of probing material, of which cross sections of the constituent nuclides are known, is irradiated at a place where the neutron dose is to be measured, and the dose is determined from the mass-spectrometric change in the probing material.

The pile oscillator is an instrument convenient for the measurement of thermal-neutron absorption cross sections. As an alternative to the above-mentioned mass-spectrometric method of neutron dosimetry, an application of the pile oscillator to large-neutron-dose measurement will be proposed: the neutron dose will be derived from the measurement of the decrement of the macroscopic absorption cross section of the probe. As far as the relative magnitude is concerned, the pile oscillator can easily measure a neutron absorption cross section with an accuracy better than 1%, providing that the sample to be measured has an adequate value of $\Sigma_a V$, where Σ_a is the neutron absorption cross section per unit volume, and V is the volume of the sample. The following will deal with the applicability of this method.

For the sake of simplicity, it is assumed that the probe consists of a single isotope and the absorption cross section of the product nucleus is negligibly small as compared with that of the original one. The pile-oscillator signals I_0 and I, each of which is proportional to the $\Sigma_a V$ of the probe before and after the irradiation, respectively, are related by

$$I = I_0 e^{-\sigma\Phi}, \quad \text{or} \quad \Phi = \frac{\log_e (I_0/I)}{\sigma},$$

where σ is the absorption cross section effective to the neutron energy spectrum at the place where the probe is irradiated, and Φ is the neutron dose which is defined, in this paper, as the time integral of the neutron flux, and has, therefore, a dimension of inverse square of length. The self-shielding in the probe is assumed to be negligible.

In Table I, some typical values of the neutron dose Φ_{I/I_0}, which is required to reduce the number of original atoms in the probe by a factor of I/I_0, are exemplified. The figure $(4.19 \pm 0.21) \times 10^{16}$ neutrons/cm², for example, means that if a piece of Gd^{157} is irradiated in a flux of 10^{12} neutrons/cm²·sec for a period of 4.2×10^4 sec (about 12 hr), 1% of Gd^{157} in the probe will disappear. In order to facilitate the measurement, it is necessary to select a probing mate-

TABLE II

THE PERCENTAGE ERRORS OF THE NEUTRON-DOSE MEASUREMENTS $\Delta\Phi/\Phi$

$\frac{\Delta\sigma}{\sigma}$	3.0%			5.0%		
$\frac{I}{I_0}$ \ $\frac{\Delta I_0}{I_0} = \frac{\Delta I}{I}$	0.5%	1.0%	2.0%	0.5%	1.0%	2.0%
0.99	70%	140%	280%	71%	140%	280%
0.95	14	28	55	15	28	55
0.90	7.4	14	27	8.4	14	27
0.80	4.4	7.0	13	5.9	8.1	14

TABLE I

THE NEUTRON DOSE, Φ_{I/I_0}, WHICH IS REQUIRED TO REDUCE THE NUMBER OF ORIGINAL ATOMS IN THE PROBE BY A FACTOR OF I/I_0[a]

Isotope	σ(2200 meters/sec)[b] (barns)	Φ_{I/I_0}		
		$\Phi_{0.99}$ (neutrons/cm²)	$\Phi_{0.90}$ (neutrons/cm²)	$\Phi_{0.50}$ (neutrons/cm²)
Gd^{157}	240,000 ± 12,000 (5%)	$(4.19 \pm 0.21) \times 10^{16}$	$(4.39 \pm 0.22) \times 10^{17}$	$(2.89 \pm 0.14) \times 10^{18}$
Sm^{149}	40,800 ± 900 (2.2%)	$(2.46 \pm 0.06) \times 10^{17}$	$(2.58 \pm 0.06) \times 10^{18}$	$(1.70 \pm 0.04) \times 10^{19}$
Cd^{113}	20,000 ± 300 (1.5%)	$(5.03 \pm 0.08) \times 10^{17}$	$(5.27 \pm 0.08) \times 10^{18}$	$(3.47 \pm 0.05) \times 10^{19}$
B^{10}	(3813)	(2.64×10^{18})	(2.76×10^{19})	(1.82×10^{20})

[a] The neutrons are assumed to be mono-energetic and have a velocity of 2200 meters/sec.
[b] D. J. Hughes and R. B. Schwartz, BNL-325, 2nd ed. (1958).