Letters to the Editors

The Depression of Thermal Neutron Flux and Density by Absorbing Foils

The depression in neutron flux resulting from the introduction of an absorbing foil into a diffusing medium has been calculated in two recent papers (*1*, *2*) and compared with experimental data on how the activation per unit mass decreases with increasing foil thickness and area. It is the purpose of this note to point out that comparisons of the calculated fluxes with the experimental data are, in fact, not strictly valid, because the latter, being measurements of the activity of $1/v$ absorbers, are measurements of neutron density. Many "flux measurements" are made with foils so thin that the distinction is not of practical importance (for a 1 mil indium foil in a void the expected depressions in thermal neutron flux and density are 4.1% and 4.9% respectively) but it should not be ignored when calculations and measurements of flux depression are being compared.

The difference in behavior of flux and density is unfortunately obscured by the assumption made in applying the, actually monokinetic, calculations to the experimental situation, namely, that the neutron energy spectrum is everywhere the same (a Maxwellian), which implies that flux and density are uniquely related and must therefore attenuate at the same rate. Although the assumption becomes increasingly accurate with decreasing foil thickness, the implication is incorrect even in the limit of zero foil thickness. It is indeed well known [e.g., Zahn (*3*)] that a thin $1/v$ absorber reduces a normally incident Maxwellian flux by the factor $[1 - (\sqrt{\pi}/2)\Sigma_0 t]$, but reduces the response of a thin $1/v$ detector to this flux by $[1 - (2/\sqrt{\pi})\Sigma_0 t]$, where t is the foil thickness and Σ_0 the cross section at the modal velocity (2200 meters/sec at 20.4°C). The difference between the two absorption coefficients is just the rate of increase of the mean neutron velocity in the foil with t, which is a finite quantity, $(4 - \pi)\Sigma_0/2\sqrt{\pi}$, even when, as t approaches zero, the actual increase in mean velocity—the spectral "hardening"—becomes vanishingly small.

It might therefore be expected that, in applying the calculations (*1*, *2*) to a thermal neutron distribution, a cross section value of $(\sqrt{\pi}/2)\Sigma_0$ should be used for calculating the flux depression, and one of $(2/\sqrt{\pi})\Sigma_0$ for the activation depression. However two effects deserve further discussion.

The first effect is spectral hardening, which is appreciable in quite thin foils when the irradiation is isotropic. For an isotropic irradiation in an infinite void, where the incident flux is the same as the unperturbed flux, the well known result, for monokinetic neutrons and a purely absorbing foil, is

$$\frac{\bar{\phi}}{\phi_0} = \frac{\bar{\rho}}{\rho_0} = \left(\frac{1}{\tau}\right)\left[\frac{1}{2} - E_3(\tau)\right] = 1 - \frac{\tau}{2}\left(\frac{3}{2} - \gamma - \log\tau\right) - \frac{\tau^2}{1.3!} + \frac{\tau^3}{2.4!} - \frac{\tau^4}{3.5!} + \cdots \tag{1}$$

where $\tau = \Sigma t$, $E_3(\tau) = \int_1^\infty u^{-3} \exp(-\tau u)\, du$, and $\gamma = 0.5772157$. ϕ denotes flux and ρ density, and the subscript zero and bar denote respectively the unperturbed value and the average over the foil. It is assumed that the foil radius is much greater than its thickness.

For a $1/v$ absorber in an isotropically incident Maxwellian flux the necessary integrals have been evaluated by Zahn (*3*) as convergent series,[1] whence

$$\frac{\bar{\phi}}{\phi_0} = 1 - \frac{\sqrt{\pi}}{2}\frac{\tau_0}{2}\left(\frac{5}{2} - \frac{3\gamma}{2} - \log 2\tau_0\right) - \frac{\tau_0^2}{6} + 0.036926\,\tau_0^3 - 0.005556\,\tau_0^4(1.751 - \log\tau_0) - 0.00206\,\tau_0^5 + \cdots \tag{2}$$

$$\frac{\bar{\rho}}{\rho_0} = 1 - \frac{2}{\sqrt{\pi}}\frac{\tau_0}{2}\left(\frac{3}{2} - \frac{3\gamma}{2} - \log\tau_0\right) - \frac{\tau_0^2}{3} + 0.047016\,\tau_0^3(1.717 - \log\tau_0) + 0.01111\,\tau_0^4 - 0.000784\,\tau_0^5(2.33 - \log\tau_0) + \cdots \tag{3}$$

Table I lists the values of the depressions $\delta_\phi (= 1 - \bar{\phi}/\phi_0)$ and $\delta_\rho (= 1 - \bar{\rho}/\rho_0)$ calculated from Eqs. (2) and (3) respectively, and values of $\delta_\phi{}^*$ and $\delta_\rho{}^*$ from Eq. 1 with $\tau = (\sqrt{\pi}/2)\tau_0$ and $(2/\sqrt{\pi})\tau_0$ respectively. The ratios δ/δ^* show that the effects of spectral hardening are not very serious, though it persists down to a very small thickness.

A more important effect is that of the diffusing medium. In contrast to the situation just considered, where no neutron can traverse the foil a second time, the incident flux is now depressed by the foil. To first order this depression is just the "missing" flux that would be produced by a source equal in strength to the neutron capture rate in the foil [see Skyrme (*4*)], and the total flux depression is, approximately, the sum of this incident flux depression and the self-shielding effect evaluated above. However, for a very thin foil, the mean velocity of this missing flux is $(\pi/4)$ times that of the unperturbed Maxwellian, and it is clear that the proper evaluation of the incident flux depression effect requires a consideration of the "thermalization" properties of the medium.

It would therefore appear that for moderators of small scattering free path, such as water, where the incident flux

[1] I am indebted to Dr. E. W. Vogt for showing that the integral arising in the calculation of the average flux reduces to $(3\sqrt{\pi}/4) - \varphi_{1/2} + d\eta/dx$, where $\varphi_{1/2}$ and η are given by Zahn.

TABLE I

VARIATION OF FLUX AND DENSITY DEPRESSION WITH FOIL THICKNESS

τ_0	δ_ϕ	δ_ϕ^*	$\delta_\phi/\delta_\phi^*$	δ_ρ	δ_ρ^*	$\delta_\rho/\delta_\rho^*$
0.0001	0.0004498	0.0004544	0.9900	0.0005554	0.0005649	0.9832
0.001	0.003478	0.003523	0.9871	0.004255	0.004350	0.9782
0.01	0.024593	0.025043	0.9820	0.029593	0.030528	0.9694
0.1	0.145359	0.149567	0.9719	0.168832	0.177251	0.9525

depression is serious for foils of quite small radius, more sophisticated calculations are required. On the other hand, the incident flux depression in graphite is an order of magnitude smaller than the self-shielding, for gold foils 0.5 in. in diameter and a few mils thick, so that for graphite, or heavy water, the present type of treatment is probably adequate.

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Ratio of Fundamental and Second Order Harmonics in Flux for Oscillator Experiments with Varying Amplitude of Reactivity Insertion

In designing oscillator rods it is important to know where the limit in terms of reactivity lies if one is to be assured of a linear response. If the reactivity is small enough, the generated flux variation δn will be small in comparison with the steady state flux level n_0, and one is allowed to make the linear assumption in the kinetic equations. In this case, for each frequency generated by the rod, only one flux component will be observed having the same frequency as the reactivity insertions by the rod.

It is easy to conceive of a situation in which the flux variations cannot be treated assuming linearity in the kinetic equations. That is, for a given rod frequency, one will observe several frequencies in the flux. Two cases can be practically visualized where this phenomenon will occur.

First, reactors which exhibit a peaking in the transfer function at power; that is, a region of tendency towards resonance due to the coupling of reactivity and physical motion in the core. In such a region the amplification will be higher and a "weak" oscillator rod might still be strong enough to generate a sizable second harmonic in the flux. Secondly, with the increasing number of reactors being built for power production, it is conceivable that one might not consider it necessary to design a special oscillator rod but would oscillate a control rod instead. This rod might be quite powerful in terms of reactivity and could generate a sizable second harmonic.

In order to analyze oscillator data with harmonic content it is necessary to obtain analytical expressions of fundamental and higher harmonics as a function of the sinusoidal reactivity insertions.[1] In most practical applications it is sufficient to obtain expressions for the fundamental and second harmonic.

In a previous paper by the author (*1*, *2*) expressions have been developed for the generalized case where nonlinear stability considerations were of prime interest.

In this note the pertinent expressions have been extracted to obtain the amplitude ratio $a = \left| X_2/X_1 \right|$ of second order harmonic and fundamental with varying amplitude K_{ex} of reactivity insertions. These equations are applicable to any reactor provided the linear frequency response is known over a range equal to twice the nonlinear range of interest. This derivation is only valid for K_{ex} considerably below prompt critical. Furthermore, the frequencies should not be too high in order to avoid the appearance of spatial modes in the flux (*3*). One nice aspect of this analysis is that all the data necessary to obtain information on nonlinear behavior is linear data.

After some straightforward algebraic manipulations with Eq. (33) (*1*) and solving for the ratio of second order harmonic to fundamental, we obtain for a zero power reactor:

$$\left|\frac{X_2}{X_1}\right| = a = \left| \frac{ZP(2j\omega)K_{ex}/2}{1 + \{ZP(2j\omega)ZP(j\omega) - 2ZP(j\omega)\mathrm{Re}[ZP(j\omega)]\}(K_{ex}/2)^2} \right| \quad (1)$$

where:

X_1 = amplitude of fundamental flux component, half peak to peak
X_2 = amplitude of second order harmonic in flux, half peak to peak

[1] In most practical cases one has an oscillator rod which does not generate a pure sinusoidal reactivity. The motion might be sinusoidal, but the reactivity is not. One will have for each mode in the input a corresponding mode of the same frequency in the flux. What one measures will be a superposition of these modes. By Fourier analysis one can obtain the individual components and one actually gets information on several frequencies with one experiment. In the extreme, one can consider a random disturbance in the reactor as a Fourier integral, and in principle obtain information on an infinite amount of frequencies. But as Dr. Bethe pointed out in an early meeting at Argonne concerned with Reactor Safety, "You put a mess in and you get a mess out." Efforts are therefore made to make the reactivity insertion as purely sinusoidal as possible, because the processing of the flux data from nonsinusoidal oscillator experiments would become unreasonably cumbersome.