On the General Solution of the Reactor Kinetic Equations

Keepin and Cox (*1*) have tabulated numerical coefficients for a general solution to the equation of reactor kinetics. They suggest that such a generalization requires that the effectiveness γ_i of delayed neutrons from the ith species compared to prompt neutrons is unity, though this is known to be a poor assumption in some reactors.

Their results can be extended, however, without further computation if we make the less restrictive assumption that the effectiveness is the same γ for all the i species. These results are expressed through coefficients for the determination of the partial fraction expansion of two functions:

$$G(s) = \left[s + \frac{\gamma}{l} \sum_i \frac{\beta_i s}{s + \lambda_i} \right]^{-1}$$

$$H(s) = \frac{\gamma \sum_i \beta_i \lambda_i (s + \lambda_i)^{-1}}{sl + s\gamma \sum_i \beta_i (s + \lambda_i)^{-1}} = G \frac{\gamma}{l} \sum_i \frac{\beta_i \lambda_i}{s + \lambda_i}$$

When γ departs from unity it is only necessary to employ an effective lifetime, l/γ, to retain exactly the form tabulated by the authors.

In the foregoing we have employed the symbol l for the lifetime or reciprocal destruction probability where the authors use Λ. We reserve Λ for the generation time or reciprocal production probability. Expressing the kinetics through equations parametric in the production rather than the destruction,

$$\frac{dn}{dt} = \frac{\rho - \bar{\beta}}{\Lambda} n + \sum_i \lambda_i \bar{c}_i + \bar{S}$$

$$\frac{d\bar{c}_i}{dt} = \frac{\bar{\beta}_i}{\Lambda} n - \lambda_i \bar{c}_i$$

where $\bar{\beta}_i = \gamma_i \beta_i$, $\bar{c}_i = \gamma_i c_i$, $\bar{S} = \gamma_s S$. This form in Λ is simpler than the form in l. Furthermore, since the control of most reactors is through changes of the destruction rather than the production probability, it is generally more accurate to treat Λ as a parametric constant than to solve for the constant l (*2*).

The Laplace transform of these equations yields the authors' result directly:

$$s\mathcal{L}n - n_0 = \frac{\mathcal{L}(\rho n)}{\Lambda} - \sum_i \frac{\bar{\beta}_i s}{\Lambda(s + \lambda_i)} \mathcal{L}n + \sum_i \frac{\lambda_i \bar{c}_{i0}}{s + \lambda_i} + \mathcal{L}\bar{S}$$

whence

$$\mathcal{L}n = \left[\frac{\mathcal{L}(\rho n)}{\Lambda} + \mathcal{L}\bar{S} \right] G + \left[n_0 + \sum_i \frac{\lambda_i \bar{c}_{i0}}{s + \lambda_i} \right] G$$

Using the authors' expansion for G leads to

$$n(t) = n_0 + \int_0^t \left[\frac{\rho n(t')}{\Lambda} + \bar{S}(t') \right] \sum_j B_j e^{(t-t')s_j} \, dt' + \int_0^t \sum_i \left(\lambda_i \bar{c}_{i0} - \frac{\bar{\beta}_i}{\Lambda} n_0 \right) e^{-\lambda_i t'} \sum_j B_j e^{(t-t')s_j} \, dt'$$

With a steady-state initial condition, $\lambda_i \bar{c}_{i0} = \bar{\beta}_i n_0 / \Lambda$, the final integral vanishes. Note that in the second formulation only the coefficients B_j enter and no use is made of the A_j, R_j of the first formulation using the lifetime l. The same generalization holds for $\gamma \neq 1$ so that the authors' tabulation of S_j and B_j is applicable to this simpler and more general solution.

Schmid (*3*) has discussed the advantages of splitting the reactivity into a time dependent term and a constant, expressing the solution in terms of the behavior of the reactor at such a constant reactivity. The whole of the present formulation is evidently a specific case when the constant is taken to be zero.

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