However, $1 - F_g \approx 1 - F_c \approx 1$, and with this replacement

$$P_g \approx P_c$$

The method is successful, then, because neutrons that enter the core from the gap are *forced* to contribute to the peaking with the proper spectrum, while the asymptotic fission rate is relatively unaffected. It is clear that the method will not work well if the ratio Σ_f/Σ_{ac} is not approximately independent of the spectrum the cross sections are averaged over. This would be the case if the fuel region were heavily poisoned with an absorber with a thermal resonance.

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Reactivity Effects of Protactinium-233 Buildup in U^{233} Fast Breeder Reactors

As part of a feasibility study of fast U^{233}–Th breeders, performed by NDA for the AEC (*1*), the effects of protactinium-233 buildup and decay on reactor control requirements have been considered. Two interesting phenomena have been studied. These are

1. buildup of reactivity after reactor shutdown
2. change in reactivity with operation

Protactinium is normally formed in the U^{233} breeder by the following chain:

$$Th^{232}(n, \gamma)Th^{233} \xrightarrow[2.33\ m]{\beta^-} Pa^{233} \xrightarrow[27.4\ d]{\beta^-} U^{233}$$

The reactivity of a shutdown core containing Pa^{233} increases steadily because the radioactive decay of Pa^{233} generates the more reactive U^{233}. Although opposite in direction, this is analogous to the xenon effects in thermal reactors. This process would place an additional control requirement on the reactor control system by inserting amount of reactivity greater than the U^{233} burnup requirements.

The magnitude of this effect has been estimated for a typical power reactor operating with an equilibrium fuel cycle (*1*). The reactor is a right cylinder 3.6 ft in diameter and length, and is surrounded by 15-in. thick Th blanket. It is cooled by sodium and fueled by U–Th metal alloy fuel elements. One fourth of the core is replaced by fresh fuel every 15 days, using a four zone radial shifting scheme. The reactor produces 760 Mw_{th}, requires a 547 kg loading of U^{233}, and operates with a breeding ratio of 1.33.

When this reactor is shut down at the end of a normal operating cycle, reactivity is inserted by decaying Pa^{233}:

$\Delta k = \Delta k_0\ (1 - e^{-\lambda t})$, where
Δk = excess reactivity released at time t
λ = decay constant of Pa^{233}
t = time after shut down
Δk_0 = potential worth of all U^{233} formed from Pa^{233}

For the particular refueling scheme studied, a decrease in reactivity of 1.76% due to U^{233} burnup occurs between refuelings. It would take 22 days of shutdown at the end of a cycle for decaying Pa^{233} to increase the reactivity by this amount. If the core remains assembled longer, the excess reactivity inserted would require additional shutdown control. The upper limit of reactivity insertion, Δk_0, is 4.1%, the net worth of all the U^{233} formed from the Pa^{233} in the core.

The previous example described the behavior of a reactor shutdown at the end of a normal operating cycle, but not yet refueled. A reactor which has been refueled and is subjected to a delay in startup, or a shutdown shortly after refueling, would also necessitate additional control requirements since there is residual Pa^{233} in the shifted fuel elements. The reactivity inserted by the decaying Pa^{233} in the residual fuel, added to the excess reactivity inserted by the fresh fuel, can cause reactivity buildup greater than 1.76% above critical.

An additional effect due to the delay time in formation of bred U^{233} in the core is the increase in net burnup reactivity change during a cycle. If the Pa^{233} had zero decay time, for example, the reactivity decrease with burnup in the reference reactor would be 1.1% as compared to an actual decrease of 1.76%. This is due to the fact that only 40% of the Pa^{233} formed by radiative capture of Th^{232} actually decays to U^{233} while in-pile.

It is also interesting to note that Pu^{239} breeders are faced with the same sort of problems due to the decay of Np^{239}. The breeding cycle in this system is

$$U^{238}(n, \gamma)\ U^{239} \xrightarrow[23.5\ m]{\beta^-} Np^{239} \xrightarrow[2.33\ d]{\beta^-} Pu^{239}$$

The half-life of Np^{239} is about $\frac{1}{12}$ that of Pa^{233}. Assuming equal power densities in U^{233} and Pu^{239} breeders, the equilibrium concentration of Np^{239} would be roughly $\frac{1}{12}$ that of Pa^{233}. Because of the small half-life of Np compared to the fuel inpile residence time, the actual concentration would be close to the equilibrium value. This is not the case in the reference U^{233} breeder, where the average Pa^{233} concentration is about $\frac{1}{2}$ its equilibrium level. The average Np^{239} concentration in a Pu^{239} breeder should therefore be about $\frac{1}{6}$ that of Pa^{233} in the U^{233} breeder, corresponding to twice the decay rate at shutdown. Up to roughly six days after shutdown, the total Np disintegrations would exceed the equivalent number of Pa disintegrations. The associated reactivity effects are a function of the distribution of bred fuel atoms and the reactivity worth of the fuel.

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Re: The Application of Statistical Methods of Analysis for Predicting Burnout Heat Flux

The statistically derived prediction method for burnout heat flux proposed by Jacobs and Merrill in the December issue (*1*) appears to contain certain features which call for further elaboration. The over-all correlation (*1*), Formula No. 8 of Fig. 12, e.g., contains 14 positive and 10 negative terms, and the burnout heat flux is obtained numerically as an often small difference between two large numbers. The associated sensitivity of the solution can be considerable for certain combinations of the variables.

Of particular interest is the effect of tube diameter on burnout heat flux when all other independent variables are held constant at their midranges, shown in Fig. 14 of reference 1. The large dependence shown completely lacks substantiation. In fact, direct experimental studies of the diameter effect have been made at the Savannah River Laboratory (*2*), and for subcooled low-pressure water flow in heated annuli with the flow gap varied from $\frac{1}{16}$ to $\frac{3}{8}$ in. and heated length fixed at 24 in., no effect on burnout was observed. Similar studies have been reported in the Russian literature (*3*) for subcooled water at 40 atmospheres flowing in a rectangular test section; as flow gap was decreased from 0.079 to 0.008 in., no effect on burnout was noted until the gap reached 0.028 in. Other Russian studies (*4*) with high-pressure water and tubular test sections of 0.157 to 0.473 in. i.d. indicated the same absence of a diameter effect. These three independent studies, made with annular, rectangular, and tubular test sections in a pressure range of 3 to 220 atmospheres, strongly suggest that a diameter effect is encountered only when the flow gap dimension becomes comparable to bubble dimensions. In the individual studies used for development of the burnout correlation of reference 1, diameter was held constant; and the apparent diameter effect, obtained by correlation of different sets of data, appears spurious.

The inapplicability of the prediction method (with the present constants) to rectangular channels is indicated by comparisons we have made between recent pertinent ORNL data (*5*) and the prediction equation. With all variables selected within the ranges used for development of the equation, the experimental values are $\backsim$1.7-fold larger than the corresponding calculated values. Whereas L and D are associated with surface area for round tubes, thereby incorporating, indirectly, enthalpy increase to the burnout point and allowing use of inlet bulk temperature, such is not the case with the length and equivalent diameter of rectangular channels.

Extrapolation of the proposed correlation (*1*) beyond the range of the data used in its development should scrupulously be avoided, as indicated by the authors. To illustrate the extreme danger inherent in carrying such an arbitrarily derived relation beyond its stated limits, a particular example may be cited. In a very high velocity (172 fps) subcooled burnout test conducted at ORNL (*6*), an experimental burnout heat flux of 17.25×10^6 Btu/hr ft^2 was attained. The conditions of this test were such that only the tube diameter was in the recommended variable range for the prediction equation, which gave a positive error of 1127%. The simpler "local phenomenon"-type equations of Gunther (*7*) and of Bernath (*8*) [a type of equation much chastised in reference (*1*)] gave errors of −30.9% and −18.9%, respectively. It would thus appear that some of the functional relations expressed by Formula No. 8 of reference 1 are seriously in error. If so, one might question the adequacy of the variable ranges cited as an application criterion, and use of the correlation in an untested region of Fig. 13 (*1*) could give erroneous results.

The authors should state where the coolant pressure is to be evaluated, since axial pressure gradients may be large enough to make site selection important. An extreme combination of the recommended variable ranges gives an isothermal ΔP of 70 psi, too large to be neglected. Similarly, a statement should be made concerning the applicability of the correlation to the bulk-boiling regime. The data in references 7 and 14 of the paper, e.g., primarily relate to tests with net steam generation, with only 8 of the tests of reference 14 conducted in the subcooled region. I assume that the method is applicable only to local-boiling burnout.

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Re: The Application of Statistical Methods of Analysis for Predicting Burnout Heat Flux—Rebuttal

Mr. Gambill has raised several questions concerning the work reported in our recent article (*1*) which appeared in this journal. The following points have been raised·

1. Where was the pressure evaluated?