LETTERS TO THE EDITOR

On the Calculation of Water Gap Peaking

Standard calculations of power peaking near water gaps employ separate hardened spectra in the adjacent gap and fuel regions (*1*). The calculations, generally, yield transients characterized by relaxation lengths that are too large, and power peakings that are too small, when compared with experiment (*2*). The origin of the discrepancy is the poor representation of the spatially dependent thermal spectrum that exists in the standard calculation. When the problem is reformulated to allow for space-energy effects, good agreement with the experiments results (*3-5*).

A particularly simple scheme is sometimes used at KAPL to obtain improved estimates of power peaking (*6*). The method consists of computing the thermal group constants in the entire reactor by everywhere averaging the cross sections over the infinite medium spectrum characteristic of the water gaps, and subsequently performing the reactor calculations with these constants. Such a scheme is especially simple with cross-section routines such as the WOXX scheme that is incorporated into the IBM-704 code CUREBO (*7*). The purpose of this communication is to note the reason for the success of the method, and to point out situations in which it might be expected to fail.

If, in slab geometry, a water gap of half width a, centered at the origin, is in an otherwise infinite, homogeneous core, then the power peaking (defined here as the ratio of the fission rate at $x = a$ to that at $x = \infty$) is given by a standard diffusion theory calculation as [see Eq. (5) of reference *3*]

$$P_s = \frac{R_c^c(1 - F)\Sigma_f^c + R_g^g F \Sigma_f^c + \langle\Sigma_f\rangle_e}{R_c^c\Sigma_f^c + \langle\Sigma_f\rangle_e} \quad (1)$$

where

R = (epithermal slowing down cross section/thermal absorption cross section)
$= \Sigma_R/\Sigma_a$
$\langle\Sigma_f\rangle e$ = epithermal fission cross section
$F = [1 + (L_g^g/D_g^g)\ (D_c^c/L_c^c)\ \coth\ (a/L_g^g)]^{-1}$ (2)

The subscripts in (1) and (2) refer to the gap (g) and core (c) regions, respectively, while the superscripts indicate the spectrum the cross sections are averaged over. For the development of (1), the epithermal flux is assumed to be position independent. L and D are the thermal diffusion length and diffusion constant, respectively.

Peakings computed from (1) are too low when compared with experiment (Table I, columns 2 and 3). When the calculation is performed using constants *everywhere* averaged over the gap spectrum, however, much improved peaking values are obtained (Table I, column 4).

The reason for the improvement may be seen by noting that the second term in the numerator of (1) represents neutrons which have been thermalized in the gap and which have subsequently diffused into the core. Since these neutrons have a much softer (i.e.; more thermal) spectrum than those neutrons which have slowed down in the core, Σ_f^c in this term should be replaced by Σ_f^g, the fission cross section averaged over the gap spectrum. On the basis of this observation, a two-thermal group scheme similar to that used by Kelber and Kier (*8*) is used in reference *3* to obtain the expression

TABLE I

COMPARISON OF EXPERIMENTAL AND COMPUTED FUEL PEAKINGS IN WRIGHT AND FEINER'S LATTICE L-7[a, b]

Gap half width (cm)	Experimental peaking[a]	P_s	P_g	P_c
0.413	2.37	1.75	2.37	2.38
0.819	3.64	2.30	3.38	3.40
1.225	4.18	2.68	4.08	4.11

[a] See reference *2*.

[b] Computed with SOFOCATE Cross sections (*9*). In all the numerical calculations, the expressions for the peaking were corrected for epithermal flux depression near the interface.

$$P_C = \frac{R_c^c(1 - F_c) + R_g^g F_g \Sigma_f^g + \langle\Sigma_f\rangle_e}{R_c^c\Sigma_f^c + \langle\Sigma_f\rangle_e} \quad (3)$$

for the peaking. F_g is obtained from (2) by replacing D_c^c/L_c^c by D_c^g/L_c^g and F_c is obtained from (2) by the replacement of D_g^c/L_g^c for D_g^g/L_g^g. Peakings obtained from (3) are given in Table I, column 5.

When, on the other hand, all the cross sections are calculated by averaging over the gap spectrum, the peaking is given by

$$P_g = \frac{R_c^g(1 - F_g)\Sigma_f^g + R_g^g F_g \Sigma_f^g + \langle\Sigma_f\rangle_e}{R_c^g\Sigma_f^g + \langle\Sigma_f\rangle_e} \quad (4)$$

an expression which seems to be quite different from (3). If, however, Σ_f and Σ_{ac} have roughly the $1/v$ dependence on energy, Σ_f/Σ_{ac} is almost independent of the spectrum the averages are taken over. In such a case

$$R_c^g\Sigma_f^g = \Sigma_{RC}(\Sigma_f^g/\Sigma_{ac}^g) \approx \Sigma_{RC}(\Sigma_f^c/\Sigma_{ac}^c) = R_c^c\Sigma_f^c$$

and (4) becomes

$$P_g = \frac{R_c^c(1 - F_g)\Sigma_f^c + R_g^g F_g \Sigma_f^g + \langle\Sigma_f\rangle_e}{R_c^c\Sigma_f^c + \langle\Sigma_f\rangle_e}$$

However, $1 - F_g \approx 1 - F_c \approx 1$, and with this replacement

$$P_g \approx P_c$$

The method is successful, then, because neutrons that enter the core from the gap are *forced* to contribute to the peaking with the proper spectrum, while the asymptotic fission rate is relatively unaffected. It is clear that the method will not work well if the ratio Σ_f/Σ_{ac} is not approximately independent of the spectrum the cross sections are averaged over. This would be the case if the fuel region were heavily poisoned with an absorber with a thermal resonance.

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Gerald P. Calame

*Knolls Atomic Power Laboratory**
Schenectady, New York

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Reactivity Effects of Protactinium-233 Buildup in U^{233} Fast Breeder Reactors

As part of a feasibility study of fast U^{233}–Th breeders, performed by NDA for the AEC (*1*), the effects of protactinium-233 buildup and decay on reactor control requirements have been considered. Two interesting phenomena have been studied. These are

1. buildup of reactivity after reactor shutdown
2. change in reactivity with operation

Protactinium is normally formed in the U^{233} breeder by the following chain:

$$Th^{232}(n,\gamma)Th^{233} \xrightarrow[2.33\ m]{\beta^-} Pa^{233} \xrightarrow[27.4\ d]{\beta^-} U^{233}$$

The reactivity of a shutdown core containing Pa^{233} increases steadily because the radioactive decay of Pa^{233} generates the more reactive U^{233}. Although opposite in direction, this is analogous to the xenon effects in thermal reactors. This process would place an additional control requirement on the reactor control system by inserting amount of reactivity greater than the U^{233} burnup requirements.

The magnitude of this effect has been estimated for a typical power reactor operating with an equilibrium fuel cycle (*1*). The reactor is a right cylinder 3.6 ft in diameter and length, and is surrounded by 15-in. thick Th blanket. It is cooled by sodium and fueled by U–Th metal alloy fuel elements. One fourth of the core is replaced by fresh fuel every 15 days, using a four zone radial shifting scheme. The reactor produces 760 Mw_{th}, requires a 547 kg loading of U^{233}, and operates with a breeding ratio of 1.33.

When this reactor is shut down at the end of a normal operating cycle, reactivity is inserted by decaying Pa^{233}:

$\Delta k = \Delta k_0\ (1 - e^{-\lambda t})$, where
Δk = excess reactivity released at time t
λ = decay constant of Pa^{233}
t = time after shut down
Δk_0 = potential worth of all U^{233} formed from Pa^{233}

For the particular refueling scheme studied, a decrease in reactivity of 1.76% due to U^{233} burnup occurs between refuelings. It would take 22 days of shutdown at the end of a cycle for decaying Pa^{233} to increase the reactivity by this amount. If the core remains assembled longer, the excess reactivity inserted would require additional shutdown control. The upper limit of reactivity insertion, Δk_0, is 4.1%, the net worth of all the U^{233} formed from the Pa^{233} in the core.

The previous example described the behavior of a reactor shutdown at the end of a normal operating cycle, but not yet refueled. A reactor which has been refueled and is subjected to a delay in startup, or a shutdown shortly after refueling, would also necessitate additional control requirements since there is residual Pa^{233} in the shifted fuel elements. The reactivity inserted by the decaying Pa^{233} in the residual fuel, added to the excess reactivity inserted by the fresh fuel, can cause reactivity buildup greater than 1.76% above critical.

An additional effect due to the delay time in formation of bred U^{233} in the core is the increase in net burnup reactivity change during a cycle. If the Pa^{233} had zero decay time, for example, the reactivity decrease with burnup in the reference reactor would be 1.1% as compared to an actual decrease of 1.76%. This is due to the fact that only 40% of the Pa^{233} formed by radiative capture of Th^{232} actually decays to U^{233} while in-pile.

It is also interesting to note that Pu^{239} breeders are faced with the same sort of problems due to the decay of Np^{239}. The breeding cycle in this system is

$$U^{238}(n,\gamma)\ U^{239} \xrightarrow[23.5\ m]{\beta^-} Np^{239} \xrightarrow[2.33\ d]{\beta^-} Pu^{239}$$

The half-life of Np^{239} is about $\frac{1}{1}\frac{1}{2}$ that of Pa^{233}. Assuming equal power densities in U^{233} and Pu^{239} breeders, the equilibrium concentration of Np^{239} would be roughly $\frac{1}{1}\frac{1}{2}$ that of Pa^{233}. Because of the small half-life of Np compared to the fuel inpile residence time, the actual concentration would be close to the equilibrium value. This is not the case in the reference U^{233} breeder, where the average Pa^{233} concentration is about $\frac{1}{2}$ its equilibrium level. The average Np^{239} concentration in a Pu^{239} breeder should therefore be about $\frac{1}{6}$ that of Pa^{233} in the U^{233} breeder, corresponding to twice the decay rate at shutdown. Up to roughly six days after shutdown, the total Np disintegrations would exceed the equivalent number of Pa disintegrations. The associated reactivity effects are a function of the distribution of bred fuel atoms and the reactivity worth of the fuel.