LETTERS TO THE EDITOR

An H_2O–D_2O Moderated Reactor

The paper, "An H_2O-D_2O Moderated Reactor," by N. P. Klug and P. F. Zweifel (*1*) contains the statement:

> "The advantages occurring from such a mixed moderator are twofold. First is the fact that D_2O "contaminated" with H_2O is much cheaper to produce than "pure" D_2O since the last stages of purification are the most expensive. Secondly, such a system may be controlled by varying the admixture of H_2O and D_2O, thus effecting a saving in control rods."

This statement indicates a significant misconception of the authors as to the costs of concentrating D_2O; the actual cost relationships are the reverse of those quoted above and should, I believe, be brought to the attention of your readers before this misconception is perpetuated.

The high cost of heavy water, produced by modern processes, is principally the result of its small natural abundance, about one part in 7,000 parts of ordinary water. The principal capital and operating costs in a heavy water plant are expended in providing and operating facilities for the first 100- to 1,000-fold concentration of D_2O. The fractionation cascade in a heavy water plant is so arranged that as the concentration increases the amount of water handled and hence the equipment size is decreased. The later stages of fractionation thus become very small, both in size and cost.

These qualitative remarks regarding the costs of heavy water are supported by the detailed actual costs presented in the paper, "Production of Heavy Water," by Bebbington and Thayer (*2*). Of the total direct cost of manufacture of heavy water (99.8% D_2O) at the Savannah River Plant, 93% is expended in bringing the concentration from natural abundance to 15% D_2O. Similarly, the investment in the facilities for concentration to 15% constitutes 96% of the total investment in process facilities. Overall, including handling charges, depreciation, and other fixed charges, the price of 15% D_2O from the Savannah River Plant would probably be no more than \$1.00 less, per pound of D_2O, than that of 99.8% D_2O; the cost of 90% D_2O would probably be about \$0.40 less than that of 99.8%.

The misconception as to the relative costs of intermediate and high concentration of D_2O may have originated in the knowledge that economically inferior processes, vacuum distillation and electrolysis, are used in the final stages of fractionation. Because the quantities of water handled are so small in these higher stages, the advantage of the simplicity and dependability of these processes can be had without appreciable cost penalty.

The control of a nuclear reactor by variation in H_2O-D_2O ratio, as suggested by Klug and Zweifel, may be feasible, but I should like to point out that if this ratio is to be varied quickly and repeatedly in a large inventory of water, a fractionating facility of very large capacity must be provided.

REFERENCES

1. N. P. KLUG AND P. F. ZWEIFEL, *Nuclear Sci. and Eng.* **7**, 541 (1960).
2. W. P. BEBBINGTON AND V. R. THAYER, *Chem. Eng. Progr.* **55**, 70 (1959).

W. P. BEBBINGTON
E. I. du Pont de Nemours & Company
Savannah River Plant
Aiken, South Carolina
Received August 31, 1960

[*Editor's Note:* This information should have been made available to the authors as a result of editorial reviews, but was not because of inadvertence in this office.]

Delayed-Neutron-Groups Number Reduction

By reducing the number of the delayed neutron groups, the tractability of the reactor kinetic equations is increased; in doing the calculation with a digital computer one saves machine time; in computing on an analog computer some integrators are saved. Especially in space-dependent kinetics these savings can be quite considerable.

Two procedures have been proposed for the determination of the constants for the smaller number of delayed neutron groups. According to the first procedure described by Skinner and Cohen (*1*), the constants are to be determined in such a way that the asymptotic behavior of the frequency response is accurately represented. For thermal fission of U^{235} and approximating with two groups, the relative abundance factors and the decay constants are

$$\begin{aligned} \beta_1/\beta &= 0.703 \qquad \lambda_1 = 0.566 \text{ sec}^{-1} \\ \beta_2/\beta &= 0.297 \qquad \lambda_2 = 0.02523 \text{ sec}^{-1} \end{aligned} \tag{1}$$

According to the second procedure (*2*) the new groups are to be introduced in such a way that in the kinetic equation

$$ln'(t) = [k(t) - 1]n(t) - \beta n(t) + \beta \int_0^\infty n(t-\tau)D(\tau)\,d\tau \tag{2}$$

the decay curve

$$D(\tau) = \sum_i \frac{\beta_i}{\beta} \lambda_i e^{-\lambda_i \tau} \tag{3}$$

is well approximated up to a time τ. We find good repre-

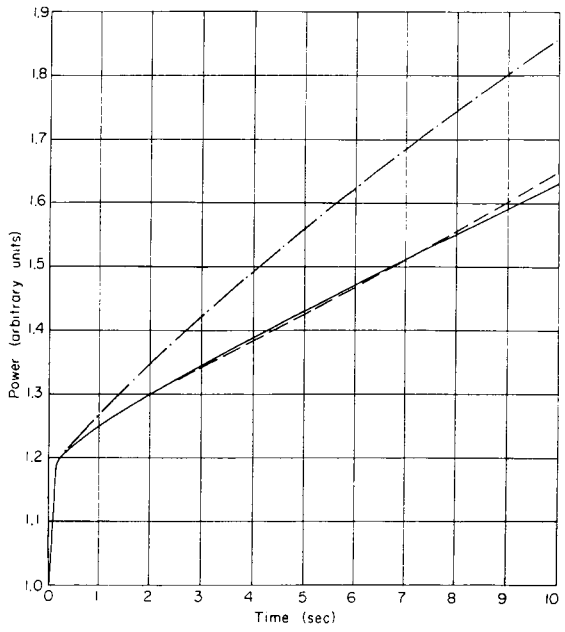

FIG. 1. Transients for step reactivity $\delta k = 10^{-3}$ and $l = 10^{-4}$ sec. KEY: —, 6 delayed neutron groups; —·—, 2 delayed neutron groups approximation according to (*1*); – – – 2 delayed neutron groups approximation according to (*2*).

sentation of the transients up to about 10 sec after the perturbation of the steady state by using the constants given below:

$$\begin{aligned} \beta_1/\beta &= 0.23 \quad & \lambda_1 &= 1.2 \ \text{sec}^{-1} \\ \beta_2/\beta &= 0.77 \quad & \lambda_2 &= 0.12 \ \text{sec}^{-1} \end{aligned} \tag{4}$$

These constants are obtained by graphical resolution of the decay curve $D(\tau)$ in the time interval $0 \leqq \tau \leqq 5$ sec.

In Fig. 1 the calculated results with the constants (1) and (4) are compared to the exact solutions using as an example a transient for step reactivity.

In representing the zero power frequency response good agreement with the exact values for very small frequencies $f < 0.003$ cps (*1*) is obtained when using the constants set (1). For the more interesting frequencies above 0.03 cps, the constants set (4) yield a good approximation (*2*). At high frequencies the frequency response generally is determined only by the total number of the delayed neutrons β as well as by the life time l of the neutrons, while the splitting into groups has little importance.

The principles of the two procedures for determining the constants sets may be combined, if three delayed neutron groups are introduced. Thus we will retain β_1 and λ_1 from the constants (4) in order that $D(\tau)$ may be approximated for smaller values of τ. The remaining two groups will be determined in such a way that $D(\tau)$ is well approximated for smaller values of τ and the asymptotic behavior of the frequency response is accurately represented for small frequencies. In this way the following constants are obtained for the thermal fission of U^{235}:

$$\begin{aligned} \beta_1/\beta &= 0{,}23 \quad & \lambda_1 &= 1.2 \quad \text{sec}^{-1} \\ \beta_2/\beta &= 0.57 \quad & \lambda_2 &= 0.165 \ \text{sec}^{-1} \\ \beta_3/\beta &= 0.20 \quad & \lambda_3 &= 0.021 \ \text{sec}^{-1} \end{aligned} \tag{5}$$

By (5), $D(\tau)$ is approximated practically in the whole range. Consequently calculations with these three groups are equal to the consideration of all six groups if the accuracy desired is not too great.

REFERENCES

1. R. E. SKINNER AND E. R. COHEN, Reduced delayed neutron group representations. *Nuclear Sci. and Eng.* **5**, 291–298 (1959).
2. A. KIRCHENMAYER, Approximation des Kurzzeitverhaltens von Reaktoren durch weniger als sechs Gruppen verspäteter Neutronen. *Nukleonik* **1**, 245–247 (1959).

A. KIRCHENMAYER

Abteilung Reaktorphysik
im Institut für theoretische und
angewandte Physik der
Technischen Hochschule,
Stuttgart, Germany

Received May 31, 1960