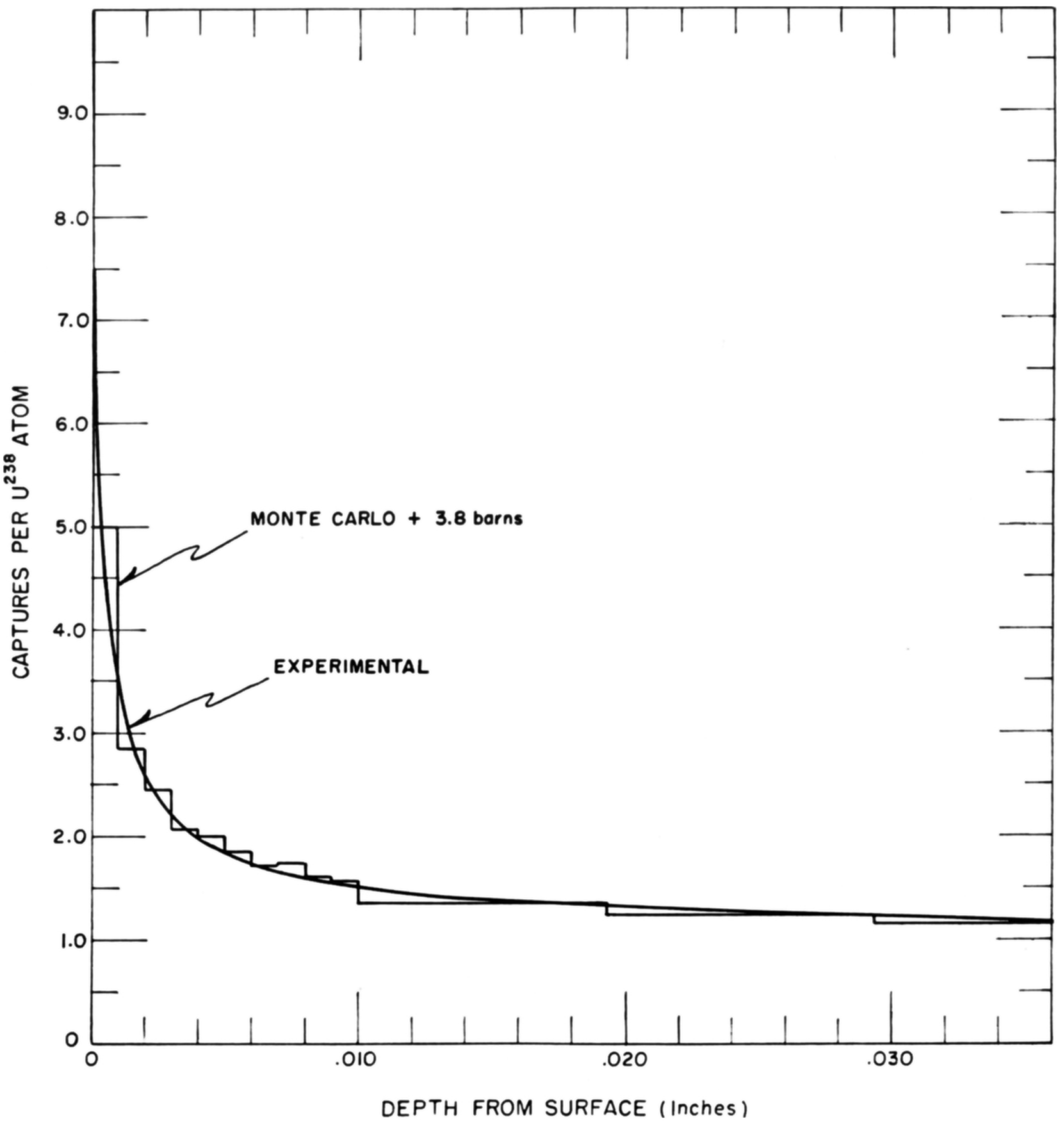

FIG. 2

gives added justification to the treatment of the higher energy resonances.

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Some Series Occurring in the Theory of the Square Lattice Cell

Cohen (*1*), Newmarch (*2*), and other authors have calculated the flux and thermal utilization in a square lattice cell: this work is of continuing interest because of its close

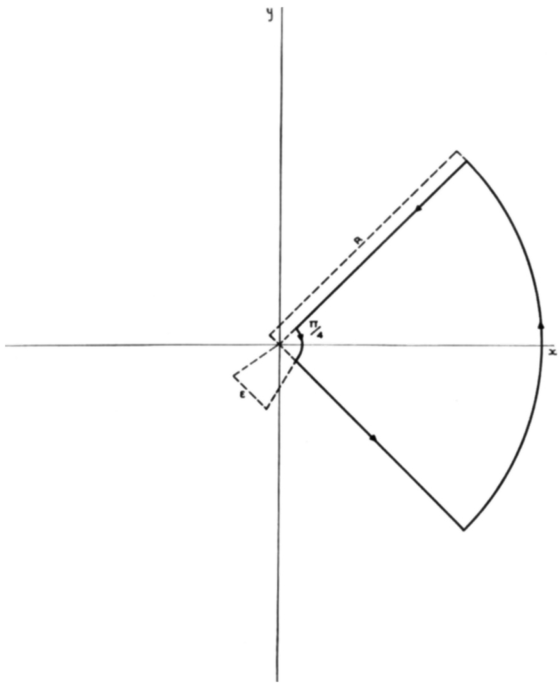

FIG. 1. The contour of integration.

connection with the Feinberg-Galanin theory of finite heterogeneous lattices. During the work, it is necessary to sum series of the form

$$C_k = \sum_{n=1}^{\infty} n^{2k-1}(\coth \pi n - 1)$$

Cohen evaluated these sums for odd values of k by comparing expressions for the flux in polar and Cartesian coordinates: he did not obtain analytical expressions for even values of k. In this note it is shown that

$$C_1 = \sum_{n=1}^{\infty} n(\coth n\pi - 1) \quad (1)$$

can be evaluated by the normal processes of analysis, and that the result obtained agrees with Cohen's. The method should generalize to other values of k.

We begin by noting that

$$\coth n\pi - 1 = 2 \sum_{r=1}^{\infty} e^{-2nr\pi}$$

so that

$$C_1 = 2 \sum_{r=1}^{\infty} \sum_{n=1}^{\infty} ne^{-2nr\pi} = \frac{1}{2} \sum_{r=1}^{\infty} \frac{1}{\sinh^2 r\pi}$$

The residue theorem therefore shows that the sum must therefore be related to the integral

$$\int \frac{\pi \cot \pi z \, dz}{\sinh^2 \pi z}$$

but the contour must be chosen with care, since the factor $1/\sinh^2 \pi z$ has double poles along the imaginary axis. The contour selected is shown in Fig. 1. It consists of arcs of circles of radii ϵ (small) and R (large) centered on the origin and extending from $\arg z = -\pi/4$ to $\arg z = +\pi/4$: these arcs are joined by straight lines. As R tends to infinity, the integral round the contour tends to $4\pi i \cdot C_1$. On the upper straight line $z = ye^{i\pi/4}$, where y is a real variable: hence on this line

$$\sinh \pi z = \sinh u \cos u + i \cosh u \sin u \qquad (u = \pi y/\sqrt{2})$$

On the lower straight line $\sinh \pi z$ is the complex conjugate of this quantity, while similar expressions may be written down for $\cot \pi z$. It follows that the sum of the integrals in the clockwise direction along the two straight lines is

$$\begin{aligned} I_1 &= +4i \int_{\epsilon\pi/\sqrt{2}}^{R\pi/\sqrt{2}} \frac{\sinh 2u \cos 2u + \sin 2u \cosh 2u}{(\cosh 2u - \cos 2u)^2} \, du \\ &= -i \left[\frac{\cosh 2u + \cos 2u}{\cosh 2u - \cos 2u} \right]_{\epsilon\pi/\sqrt{2}}^{R\pi/\sqrt{2}} \\ &= -i \left\{ 1 + 0\,(e^{-R\pi/\sqrt{2}}) - \frac{1}{\epsilon^2\pi^2} + 0(\epsilon^2) \right\} \end{aligned}$$

The integral round the big arc is exponentially small if R is large, while on the small circle

$$\frac{\pi \cot \pi z}{\sinh^2 \pi z} = \frac{1}{\pi^2 z^3} - \frac{2}{3z} + 0(z)$$

so that the sum of the integrals taken clockwise round the arcs is

$$I_2 = 0(e^{-R\pi/\sqrt{2}}) - i \left\{ \frac{1}{\pi^2\epsilon^2} - \frac{\pi}{3} + 0(\epsilon^2) \right\}$$

Then

$$C_1 = \frac{1}{4\pi i} (I_1 + I_2) = \frac{1}{12} - \frac{1}{4\pi} \quad (2)$$

as ϵ is allowed to tend to zero and R to infinity. Equation (2) agrees with Cohen's result.

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