has been adopted. The theoretical curves are calculated from Eq. (9), with the only requirement that the theoretical curve (rod diameter 8 mm) should agree with the experimental value for the point $r = 0$.

One further point of evidence comes from a comparison with Monte-Carlo calculations made by Morton (2), Fig. 3. In this case the analytic curve, without volume contribution, is shown. The Monte-Carlo results are derived under more realistic premises: Doppler-broadening of resonances, inclusion of resonance scattering, higher order collisions with energy degradation. The results, nevertheless, show the same trend as the "exact" distribution.

In the immediate vicinity of the surface the radial function no longer is independent from resonance parameters. For cylindrical rods one obtains

$$A(x) = \Sigma_0 \frac{\Gamma/2}{E_0} \cdot S^* \left\{ \pi F_2 \left(\frac{\Sigma_0 x}{2} \right) + \frac{1.8285}{\sqrt{\Sigma_0 R}} \right\} \tag{15}$$

in which $x = R - r$ is of the order $1/\Sigma_0$ and $F_2(\xi)$ is defined by

$$F_2(\xi) = \int_1^\infty e^{-\xi t} I_0(\xi t) \frac{dt}{t^2} \tag{16}$$

Männer and Springer (5) recently investigated the activation of plane resonance foils with similar methods and showed good correspondence with experimental results.

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A Note on the Perturbation Method in Neutron Transport Theory

In this note the perturbation formula of neutron transport theory (1–3) is derived by the method of ordinary perturbation theory used in quantum mechanics. Using the standard methods, formulas for higher order perturbations may be written down immediately.

As is well known the Boltzmann equation

$$\begin{aligned} \mathbf{\Omega} \text{ grad } \psi(\mathbf{v}, \mathbf{r}) + \alpha(\mathbf{v}, \mathbf{r})\psi(\mathbf{v}, \mathbf{r}) &= q(\mathbf{v}, \mathbf{r}) \\ \gamma\, q(\mathbf{v}, \mathbf{r}) &= \int \beta(\mathbf{v}, \mathbf{v}'; \mathbf{r})\psi(\mathbf{v}'; \mathbf{r})\, d^3v' \end{aligned} \tag{1}$$

can be converted into an integral equation

$$\begin{aligned} \psi(\mathbf{v}, \mathbf{r}) &= \int K^{(\alpha)}(\mathbf{v}; \mathbf{r}, \mathbf{r}')q(\mathbf{v}, \mathbf{r}')\, d^3r' \\ \gamma q(\mathbf{v}, \mathbf{r}) &= \int \beta(\mathbf{v}, \mathbf{v}'; \mathbf{r})\psi(\mathbf{v}', \mathbf{r})\, d^3v' \end{aligned} \tag{2}$$

which in turn can be expressed in operator form

$$\begin{aligned} \psi = K^{(\alpha)}q \qquad \gamma q &= \beta\psi \\ \text{or} \qquad \gamma\psi &= K\beta\psi \end{aligned} \tag{3}$$

ψ being the directional flux, q the emission density, α the total cross section, $\beta(\mathbf{v}, \mathbf{v}'; \mathbf{r})$ the "transfer cross section" of neutrons of velocity $\mathbf{v}$ into a velocity $\mathbf{v}'$ lying in the element d^3v of velocity space. The eigenvalue γ which is the multiplication in one "scattering generation" is introduced instead of the reactivity. The kernels $K^{(\alpha)}$ and β possess the following symmetry properties:

$$\begin{aligned} K^{(\alpha)}(\mathbf{v}; \mathbf{r}, \mathbf{r}') &= K^{(\alpha)}(-\mathbf{v}; \mathbf{r}', \mathbf{r}) \\ \beta(\mathbf{v}, \mathbf{v}'; \mathbf{r}) &= \beta(-\mathbf{v}, -\mathbf{v}', \mathbf{r}) \end{aligned} \tag{4}$$

In order to prove the perturbation formula we further transform the Boltzmann equation into a new form. Defining

$$\beta^{(0)}(\mathbf{v}, \mathbf{v}'; \mathbf{r}) = (1/\gamma)\beta(\mathbf{v}, \mathbf{v}'; \mathbf{r}) - \alpha(\mathbf{v}, \mathbf{r})\, \delta_{vv'} \tag{5}$$

the Boltzmann Eq. (1) becomes

$$\begin{aligned} \mathbf{\Omega} \text{ grad } \psi(\mathbf{v}, \mathbf{r}) &= q^{(0)}(\mathbf{v}, \mathbf{r}) \\ q^{(0)}(\mathbf{v}, \mathbf{r}) &= \int \beta^{(0)}(\mathbf{v}, \mathbf{v}'; \mathbf{r})\psi(\mathbf{v}', \mathbf{r})\, d^3v' \end{aligned} \tag{6}$$

which, in turn, can be written in operator form

$$\psi = K^{(0)}q^{(0)} \qquad q^{(0)} = \beta^{(0)}\psi \tag{7}$$

or

$$\psi = K^{(0)}\beta^{(0)}\psi \qquad \text{and} \qquad q^{(0)} = \beta^{(0)}K^{(0)}q^{(0)}$$

We generalize these equations by introducing an eigenvalue ϵ,

$$\begin{aligned} K^{(0)}\beta^{(0)}\psi_\epsilon &= \epsilon\psi_\epsilon \\ \text{and} \qquad & \\ \beta^{(0)}K^{(0)}q_\epsilon^{(0)} &= \epsilon q_\epsilon^{(0)} \end{aligned} \tag{8}$$

The physical solution for the flux ψ corresponds to the eigenvalue $\epsilon = 1$, all other solutions correspond to eigenvalues smaller than one. Two further equations with the same eigenvalues are formed with the transposed operators. Considering the symmetries (4), it is readily seen that the transposed of the equation for the emission density $q_\epsilon^{(0)}$ is equivalent to the Boltzmann equation (1) with β replaced by $(1/\epsilon)\tilde{\beta}$, α by $(1/\epsilon)\alpha$ and all the velocities reverted, i.e., the adjoint Boltzmann equation. The solutions $\psi_\epsilon^+(-\mathbf{v}, \mathbf{r})$ of the adjoint Boltzmann equation being eigenvectors of $\widetilde{(\beta^{(0)}K^{(0)})}$ are orthogonal to the eigenvectors $q_\epsilon^{(0)}$ of (βK) belonging to different eigenvalues. This last fact can now be used to formulate perturbation theory in a straightforward manner.

$K^{(0)}$ is the matrix $K^{(\alpha)}$ for $\alpha = 0$ and is completely independent of the material properties of the system. Any perturbation in the material properties α, β and the multiplication factor γ is expressed as a variation of $\beta^{(0)}$. The usual first-order perturbation formula is

$$\delta\epsilon = (\psi^{+}\cdot(\delta\beta^{(0)})K^{(0)}q^{(0)}) \tag{9}$$

The scalar product $(\psi^{+}\cdot\varphi)$ of ψ^{+} with a vector $\varphi(\mathbf{v}, \mathbf{r})$ is defined as

$$\iint \psi^{+}(-\mathbf{v}, \mathbf{r})\varphi(+\mathbf{v}, \mathbf{r})\, d^3v\, d^3r$$

in the physical case $\epsilon = 1$, and therefore we equate $\delta\epsilon$ to zero. Using (7) and (5), the above expression for $\delta\epsilon$ may be rewritten as follows

$$\begin{aligned}\delta\epsilon = 0 &= \iiint \psi^{+}(-\mathbf{v}, \mathbf{r})(\delta\beta^{(0)}(\mathbf{v}, \mathbf{v}'; \mathbf{r}))\psi(\mathbf{v}', \mathbf{r})\, d^3v\, d^3v'\, d^3r \\ &= \iiint \psi^{+}(-\mathbf{v}, \mathbf{r})\delta(\beta(\mathbf{v},\mathbf{v}'; \mathbf{r})/\gamma)\psi(\mathbf{v}',\mathbf{r})\, d^3v, d^3v'\, d^3r \\ &\quad - \iint \psi^{+}(-\mathbf{v}, \mathbf{r})\psi(\mathbf{v}, \mathbf{r})\delta\alpha(\mathbf{v}, \mathbf{r})\, d^3v\, d^3r\end{aligned} \tag{10}$$

In general the variation $\delta\alpha$ consists of a variation $\delta\Sigma$ of the cross sections and a variation $\delta\lambda/v$ of the fictitious absorption cross section λ/v due to a time variation $\exp \lambda t$ of the system (*4*).

In the same way any number of terms in the perturbation series for $\epsilon(= 1)$ may be written down immediately and equated to zero. Thereby we get relations between the changes in α, β and γ. For example the second-order perturbation formula

$$\delta\epsilon = (\psi_1{}^{+}\cdot\delta\beta^{(0)}K^{(0)}q_1^{(0)}) + \sum_{\epsilon\neq 1} \frac{(\psi_1{}^{+}\cdot\delta\beta^{(0)}K^{(0)}q_\epsilon^{(0)})(\psi_\epsilon{}^{+}\cdot\delta\beta^{(0)}K^{(0)}q_1^{(0)})}{1-\epsilon} \tag{11}$$

yields the relation

$$\begin{aligned}\delta\epsilon = 0 = &\left(\frac{\delta\gamma}{\gamma}\right)^2 \sum_{\epsilon\neq 1} \frac{1}{1-\epsilon}\left\langle 1 \left| \frac{\beta}{\gamma} \right| \epsilon\right\rangle\left\langle \epsilon \left| \frac{\beta}{\gamma} \right| 1\right\rangle \\ &- \left(\frac{\delta\gamma}{\gamma}\right)\left\{\left\langle 1 \left| \frac{\beta}{\gamma} \right| 1\right\rangle + \sum_{\epsilon\neq 1}\frac{1}{1-\epsilon}\left[\left\langle 1 \left| \frac{\beta}{\gamma} \right| \epsilon\right\rangle\left\langle \epsilon \left| \left(\frac{\delta\beta}{\gamma} - \delta\alpha\right) \right| 1\right\rangle \right.\right. \\ &\left.\left. + \left\langle 1 \left| \left(\frac{\delta\beta}{\gamma} - \delta\alpha\right) \right| \epsilon\right\rangle\left\langle \epsilon \left| \frac{\beta}{\gamma} \right| 1\right\rangle\right]\right\} \\ &+ \left\{\left\langle 1 \left| \left(\frac{\delta\beta}{\gamma} - \delta\alpha\right) \right| 1\right\rangle \right. \\ &\left. + \sum_{\epsilon\neq 1}\frac{1}{1-\epsilon}\left\langle 1 \left| \left(\frac{\delta\beta}{\gamma} - \delta\alpha\right) \right| \epsilon\right\rangle\left\langle \epsilon \left| \left(\frac{\delta\beta}{\gamma} - \delta\alpha\right) \right| 1\right\rangle\right\}\end{aligned} \tag{12}$$

where

$$\begin{aligned}\langle \epsilon' \mid \delta\alpha \mid \epsilon''\rangle &= \iint \psi_{\epsilon'}^{+}(-\mathbf{v}, \mathbf{r})\psi_{\epsilon''}(\mathbf{v}, \mathbf{r})\delta\alpha\cdot d^3v\, d^3r \\ \left\langle \epsilon' \left| \frac{\beta}{\gamma} \right| \epsilon''\right\rangle &= \frac{1}{\gamma}\iint \psi_{\epsilon'}^{+}(-\mathbf{v}, \mathbf{r})q_{\epsilon''}(\mathbf{v}, \mathbf{r})d^3v\, d^3r \\ \left\langle \epsilon' \left| \frac{\delta\beta}{\gamma} \right| \epsilon''\right\rangle &= \frac{1}{\gamma}\iiint \psi_{\epsilon'}^{+}(-\mathbf{v}, \mathbf{r})\delta\beta(\mathbf{v}, \mathbf{v}'; \mathbf{r})\psi_{\epsilon''}(\mathbf{v}', \mathbf{r})\, d^3v\, d^3v'\, d^3r\end{aligned} \tag{13}$$

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Errata

Volume 7, Number 3, March 1960, in the article by Morton R. Fleishman and Harry Soodak entitled "Methods and Cross Sections for Calculating the Fast Effect," pp. 217–227:

Page 224, Table II change:

σ_{1t} for U^{238} from 4.52 to 4.541
σ_{1c} for U^{238} from 0.054 to 0.032
σ_{1t} for UO_2 from 7.77 to 7.796
σ_{1c} for UO_2 from 0.099 to 0.077