LETTERS TO THE EDITOR

## Development of Flow in a Loop after Pumping Startup

In a previous study (*1*), an analytical method was proposed for the determination of flow coastdown in a loop. It was found that the flow coastdown could be approximated quite well by using an approach which did not require a knowledge of specific pump characteristics. This method is being extended here to the related problem of pump startup at constant applied torque.

The system, as shown in Fig. 1 of reference 1, consists of a centrifugal pump in a closed loop made up of pipes and channels of various lengths and cross sections. The detailed derivation of the pump and flow equations is contained in reference 1 and is not repeated here. The analysis is based on the simplifying assumption that the impeller torque and pump head are proportional to the square of the pump speed. The generalized coastdown equations for the pump and for the fluid in the loop are

$$\frac{d\Omega}{dT} + \alpha\Omega^2 = 0 \tag{1}$$

$$\frac{dQ}{dT} + Q^2 = \Omega^2 \tag{2}$$

All of the variables in the above equations are normalized. $\Omega$ is the ratio of transient to steady-state pump speed, $T$ is the ratio of elapsed time to the loop half-time, $\alpha$ the ratio of the energy in the fluid to the effective energy stored in the pump, and $Q$ the ratio of the transient flow to steady-state flow. The loop half-time, $t_{1/2}$, is the time required for the flow in a loop to diminish to one-half the initial flow when there is no pump in the loop.

In the startup problem, the flow equation (2), remains the same. Equation (1), however, is no longer homogeneous, as in the coastdown problem, since now there is an external applied torque. The magnitude of the applied torque, furnished by the electric motor, depends generally upon the motor characteristics and associated startup network. For simplification, it is assumed that we are dealing with a dc motor and that the external startup network controls both the field and armature current to produce a more or less constant torque during speed change. For a shunt wound motor, for example, this would mean a proportionate decrease in the field current with increase in armature current to maintain a constant product of field flux and armature current. However, in common practice, the field flux remains essentially constant and resistance is cut out of the armature circuit with increasing speed and increasing counter emf, or the armature current terminal voltage is varied, to keep the armature current more or less constant. This may be done in a stepwise manner, but it will produce, on the average, a constant startup torque. The pump startup equation, assuming a constant applied torque, is

$$J(d\omega/dt) + C\omega^2 = C\omega_0^2 = M_0 \tag{3}$$

where $J$ is the mass polar moment of inertia of the mass attached to the impeller shaft and entrained fluid, $\omega$ the circular speed of the pump, $\omega_0$ the final steady-state speed of the pump, $t$ the elapsed time, and $M_0$ the steady applied torque. The pump halftime which is the time required for the pump to reduce its speed, during coastdown, to one-half its original speed, assuming a constant pump characteristic, is sometimes called the air startup time. If the startup took place in air rather than water there would be no retarding impeller torque and Eq. (3) would be

$$J(d\omega/dt) = C\omega_0^2$$

yielding

$$t = (J\omega/C\omega_0^2)$$

When $\omega$ is set equal to $\omega_0$, the startup time is obtained. From the above

$$t_{\text{startup in air}} = \tau_{1/2} = J/C\omega_0$$

Equation (3) is now normalized and becomes

$$d\Omega/dT + \alpha\Omega^2 = \alpha \tag{4}$$

Taking into account the initial condition $\Omega(0) = 0$, the solution of Eq. (4) yields

$$\Omega = \tanh \alpha T \tag{5}$$

Equation (5) is now substituted into Eq. (2) to give the loop flow equation

$$dQ/dT + Q^2 = \tanh^2 \alpha T \tag{6}$$

The initial condition $Q(0) = 0$, applies to this equation. Equation (6) is the analytically derived equation describing the startup flow. The more exact solution of the problem, taking into account the pump characteristics, may be obtained from the following pump and flow equations:

$$d\Omega/dT + m\alpha = \alpha \qquad \Omega(0) = 0 \tag{7}$$

and

$$dQ/dT + Q^2 = h \qquad Q(0) = 0 \tag{8}$$

In these equations $m$ is the ratio of the transient torque to steady-state torque and $h$ is the ratio of transient pump head to steady-state pump head. The quantities $m$ and $h$ are the common pump operating characteristics and are often shown as a function of the ratio of pump speed to discharge velocity.

The curves representing the solution of Eq. (6) are

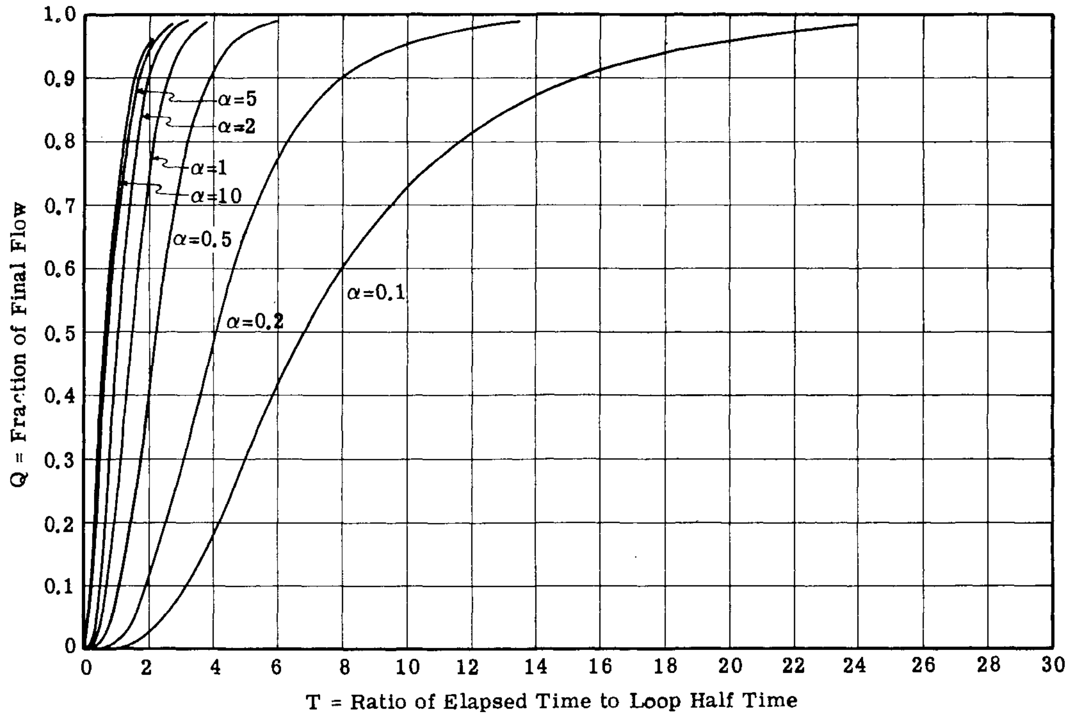


FIG. 1. Analytical startup curves.

plotted in Fig. 1. To compare these results with those obtained by the use of pump characteristic curves, Eqs. (7) and (8) were applied to a single suction Voith pump and a double suction DeLaval pump. The characteristics of these pumps are represented by the polynomials given in Table 1 of reference 1.

The curves obtained from the analytical solution and from the characteristic curve solutions show similar flow startup patterns. For small values of the half-time ratio $\alpha$ the correspondence is very close. This is to be anticipated since low values of $\alpha$ indicate high inertia impellers which require a longer time to be brought up to steady-state operating speed. During this period, the fluid is brought up slowly to steady-state flow, with the result that there is no large change in the ratio of speed of pump to velocity of flow. The constant-characteristic-based analytical solution assumes that this ratio remains constant. Therefore, for small values of $\alpha$ the condition is closely satisfied.

Large values of $\alpha$ imply the presence of impellers with low inertia. During constant torque startup, such impellers would approach steady-state operating speed much more rapidly than the flow would approach steady-state flow. This produces a higher pressure across the pump than there would be if the flow increased proportionately with pump speed. The higher head across the pump would then accelerate the flow and accomplish the startup transient more rapidly than the analytical constant-characteristic solution would indicate. This was observed in comparing the analytical curves for high $\alpha$ with the true characteristic curves. From a practical point of view this discrepancy is not important, since with very light impellers the flow is established so rapidly that it is substantially a step jump. In view of this, the curves in Fig. 1 should give a fair estimate of the startup flow transient under constant impeller torque.

REFERENCE

1. D. BURGREEN, Flow coastdown in a loop after pumping power cutoff. *Nuclear Sci. and Eng.* **4**, 306 (1959).

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## Spatial Distribution of Resonance Absorption in Fuel Elements

The spatial distribution of resonance absorption in fuel elements is a problem of high importance in reactor calculations. Usually most investigators (*1*, *2*) have used Monte-Carlo methods which are very time-consuming and do not permit general conclusions. In this note an analytic approach to the problem is presented which leads to simple results and may well be generalized for similar applications.

The energy variation of cross section for strong resonance absorbers such as $U^{238}$ may be described by a succession of Breit-Wigner resonances. Now, if it is possible to evaluate the spatial distribution of resonance absorption which may be attributed to some isolated resonance, one may expect the total absorption to be describable in terms of contributions from individual resonances.

Let us consider a cylindrical lump, immersed in moderator, which contains a strong resonance absorber uniformly