$$q(z, E) = \frac{1}{2} l(E)M(E)E_3\left(\frac{z}{l(E)}\right) + \frac{1}{2}\int_0^\infty dz' \int_0^\infty dE'E_1\left(\frac{|z-z'|}{l(E)}\right)\Sigma(E'\to E)q(z',E') \tag{2}$$

In terms of $q(z, E)$ the extrapolation distance is given by

$$z_0 = \frac{3}{2\bar{l}}\left[\frac{\overline{l^2}}{4}+\frac{I_1}{2}\right]$$

where

$$I_1 = \int_0^\infty\int_0^\infty\int_0^\infty dE\ dE'\ dz l(E)E_3\left(\frac{z}{l(E)}\right)\Sigma(E'\to E)q(z, E') \tag{3}$$

$$\bar{l} = \int l(E)M(E)\ dE$$

and

$$\overline{l^2} = \int l^2(E)M(E)\ dE$$

By use of the integral equation (2) satisfied by $q(z, E)$, and the detailed balance condition, it can be shown that a stationary expression for I_1 is given by

$$'I_1' = \tilde{I}_1{}^2/\tilde{I}_2 \tag{4}$$

where

$$\tilde{I}_2 = \int_0^\infty\int_0^\infty\int_0^\infty dE\ dE'\ dz\tilde{q}(z, E)\Sigma(E'\to E)[M(E)]^{-1} \times\left[\tilde{q}(z, E') - \frac{1}{2}\int_0^\infty\int_0^\infty dE''dz'E_1\left(\frac{|z-z'|}{l(E')}\right)\Sigma(E''\to E')\tilde{q}(z', E'')\right]$$

and $\tilde{I}_1$ is the integral given in (3) with $q(z, E)$ replaced by the trial function $\tilde{q}(z, E)$. If $\tilde{q}(z, E)$ is chosen as the solution of (2), the expression (4) for I_1 will be maximized.

The simplest trial function is the asymptotic solution $\tilde{q}(z, E) = CM(E)$. The integrals for this trial function are the same that occur in the one-velocity problem and give

$$z_0 = \frac{3}{8}\frac{\overline{l^2}}{\bar{l}} + \frac{1}{3}\bar{l} \tag{5}$$

For constant mean free path this reduces to 0.7083 l, which is only 0.3% smaller than the exact value. The prescription given by (5) is, therefore, likely to be quite good if the variation of the mean free path over the thermal energy region is not too great.

It is of some interest to compare the extrapolation distance and the thermal diffusion coefficient in a purely thermal neutron spectrum. For isotropic scattering the thermal diffusion coefficient is given by $D = \bar{l}/3$. For constant mean free path, we therefore have $z_0 = 2.13D$. As an example of a varying mean free path we consider the case of water where the transport cross section varies approximately as $1/v$ (*9*). We will assume that the isotropic scattering results can be carried over by replacing $l(E)$ by the transport mean free path. [This is known to be accurate for the diffusion coefficient (*2*), and for the extrapolation distance in the case of constant cross section (*1*).] The relation between extrapolation distance and diffusion coefficient for a $1/v$ cross section, as obtained from (5), is

$$z_0 = 2.13D\left[\frac{8}{17}+\frac{9}{17}\left(\frac{32}{9\pi}\right)\right]$$

The extrapolation distance for water is, therefore, approximately 7% greater than would be obtained from the diffusion coefficient and the assumption of constant cross section.

There is, however, a great deal of the physics of the problem omitted by the use of the asymptotic solution as trial function. No account is taken of the departure from an equilibrium velocity distribution near the boundary. This departure is caused by the preferential leakage of neutrons of longer mean free path, and depends on the ability of collisions with the moderator to restore equilibrium. These effects can be considered, in principle, by using a trial function which goes to the correct asymptotic form at large distances but includes a non-Maxwellian transient near the boundary. At this point the analogy with the constant cross section case breaks down, and the evaluation of the integrals becomes extremely complicated. Since the results to date were felt to be of sufficient interest to be put on record, this letter has been written.

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5. Hurwitz, Nelkin, and Habetler, *Nuclear Sci. and Eng.* **1,** 280 (1956). The detailed balance condition is discussed in appendix A.
6. All integrations involving E_n functions carried out in this report are the same as in the one velocity case. A table of integrals involving $E_n(x)$ is available in the Canadian report MT-131 by J. Le Caine.
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Volcanic Energy Resources

One of the possibilities for long term world energy production is the earth's internal heat. Relatively little is known of its total reserves, or even what fraction is being replenished by radioactivity. An estimate can be made, however, of the average amount of energy being wasted annually through the high-temperature process of lava emission. This may be only a small fraction of the total energy production. A value of 0.8 km^3 of lava per year (Sapper 1927) is still considered (*1*) the best estimate, based on production since 1500 A.D. Taking an estimate of 2000°F at emission, a density of 3, and an average atomic weight of

25, this corresponds to the fission of about 42 metric tons of uranium or thorium per year. Forty tons per day is given (*2*) as the rate corresponding to twenty times the world's present energy consumption. Thus, the energy wasted in volcanism is about 6% of the world's current consumption.

The geographic distribution of volcanic energy is generally complementary to that of fossil fuels, and has already led to competitive electric power generation from fumaroles in a few places. The Italian installations at Larderello have been recently described by Schnell (*3*). About 3000 metric tons of natural steam per hour are currently used there in the production of approximately 2×10^9 kw-hr of electricity per year. Large scale use of volcanic energy might eventually be of value in locally stabilizing the earth's crust and increasing property values in volcanic regions.

The development of volcanic energy resources might prove an attractive alternative to fission reactor development in several regions such as Italy, Japan, Mexico, Alaska, and Hawaii during the next several decades.

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Criticality Study on TREAT Reactor-Cause of Excess Boron Impurity in Graphite

Criticality calculations for the TREAT reactor, assuming no slots and no test hole, and using a modified Fermi-Age Theory which allows for epithermal absorption and fission, indicated a critical radius of 59 cm. At the time of startup, TREAT became critical at a radius of 67.8 cm. The discrepancy between the calculated and measured values was found to be due to boron impurity in the graphite in excess of 1.0 ppm allowed in the original specification. The results of spectrochemical and chemical tests on 50 random samples indicated an average boron impurity in the core of 7.6 ppm with an average deviation of 1.6.

An effort was made to determine the origin of this boron impurity in order to ascertain if the value of 7.6 ppm was representative of the core, and also in order possibly to prevent the recurrence of this kind of graphite contamination. The reason for the excess of boron, determined by the Metallurgy Division, is as follows:

The fuel tubes used in the TREAT reactor were baked in borated (nominally 2 w/o boron) stainless steel separators, covered with loose graphite powder. Impurity tests of graphite samples were made in a repeated experiment simulating baking operations. After cooling, analysis was made of the graphite samples which were in contact with the borated steel surface and at distances $1\frac{1}{2}$ and 3 in. away from the borated steel surface. The corresponding contents of boron impurities were 20, 5, and 4 ppm, respectively. This seemed to indicate that at the baking temperature, some of the boron atoms that were not in stable solution in the steel, migrated from it, and were transferred to the graphite in the fuel element (*1*).

REFERENCE

1. For more details, see H. P. Iskenderian, Post-criticality analysis of the TREAT reactor. ANL-6115 (1960).

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Errata

Volume 7, Number 2, February 1960, in the article by Jeffery Lewins entitled, "The Use of the Generation Time in Reactor Kinetics," pp. 122–126:

p. 126, eq. (15): for a_i read s^*a_i

p. 126 eq. (18): for t^{**} read t^*