This method essentially fits a polynomial of order n to $n + 1$ points, and is quite simple and rapid if the calculations are properly systematized [2]. However, the precision of the interpolated result at x can be improved upon if the function being interpolated is indeed a polynomial, preferably of low order. It will now be shown that $xB^2(x)$ is approximately parabolic in x and hence, capable of more accurate interpolation than is $B^2(x)$.

Assume that resonance escape, thermal utilization, and thermal migration area are the only lattice constants whose changes with x are typically the most significant and can be approximated as

$$p = \exp(-c_1/x) \tag{3}$$

$$f = 1/(1 + c_1 x) \tag{4}$$

$$L^2 = L_0^2(1 - f) = L_0^2[1 - (1 + c_2 x)^{-1}] \tag{5}$$

then in simple one group theory

$$B^2 = \frac{\epsilon\eta p f - 1}{L^2 + \tau} \tag{6}$$

$$\frac{dB^2}{dx} = \frac{k_\infty c_1}{M^2} \cdot \left(\frac{1}{x^2} - \frac{1}{x_m^2}\right) \tag{7}$$

The maximum buckling is at $x = x_m$, where

$$\frac{1}{x_m^2} = \left(\frac{L_0^2 B^2}{k_\infty} + \frac{1}{f}\right)\frac{c_2 f^2}{c_1} \tag{8}$$

Since c_1 and c_2 typically are weakly dependent on x, and since other lattice constants are slowly varying compared to $x^{-2} - x_m^{-2}$ near x_m, an integration of Eq. (7) yields

$$B^2 = -A_1 x + A_2 - (A_3/x) \tag{9}$$

The A_i are approximately constant. Therefore, xB^2 is approximately parabolic in x near x_m.

Figure 1 shows an example of the extent to which xB^2 is parabolic by testing for linearity in $d(xB^2)/dx$ from rigorously calculated bucklings [3]. This almost linear behavior in the derivative has been found among many other calculated and experimental bucklings.

The above supposition that

$$B^2(x) = \frac{1}{x} \sum_{i=0}^{n} L_i\, x_i\, B^2(x_i) \tag{10}$$

is more accurate than Eq. (1) by virtue of the parabolic nature of $xB^2(x)$ has indeed been found to be the case in practice. Table I compares Eqs. (1) and (10) as interpolation techniques for obtaining B^2 (1.5) and B^2 (3) from the calculated bucklings, B^2 (1), B^2 (2), and B^2 (4). It is seen that Eq. (10) is a few times more accurate than Eq. (1). Also, it seems to give surprisingly good interpolated values when compared with the calculated bucklings at $x = 1.5$ and 3, in spite of the factor of 2 interval size used for interpolation.

It is believed that Eq. (10) can be useful in connection with both theory and experiment. For the former, parameter studies made in reactor design and evaluation could conceivably involve fewer buckling calculations without loss of appreciable precision. The most precise interpolation between experimental points should be based on theoretical calculations of buckling. Lacking the latter, Eq. (10) should prove useful. This is especially the case for high precision experimental points, which might otherwise have a "draftman's eye" type curve drawn through them.

TABLE I

COMPARISON OF INTERPOLATION TECHNIQUES FOR BUCKLINGS

Moderator to fuel ratio, x	Calculated B^2 in μB, [3]	Errors of interpolated bucklings in μB: Ordinary linear interpolation using x_i = 1, 2	Ordinary linear interpolation using x_i = 2, 4	Interpolation by Eq. (1) using x_i = 1, 2, 4		Interpolation by Eq. (10) using x_i = 1, 2, 4	
1	2671[a]						
1.5	4791[a]	−568		−305		134	
2	5774[a]						
3	6256[a]		−541		513		−190
4	5655[a]						
1	1022[b]						
1.5	2830[b]	−522		−281		121	
2	3593[b]						
3	3757[b]		−488		477		−166
4	2945[b]						
1	1553[c]						
1.5	3523[c]	−655		−361		128	
2	4183[c]						
3	3729[c]		−440		734		−49
4	2394[c]						

[a] This set of bucklings used 0.387 in. diameter, 1.3% enriched uranium rods.

[b] This set of bucklings used 0.387 in. diameter, 1.0% enriched uranium rods.

[c] This set of bucklings used 0.600 in. diameter, 1.0% enriched uranium rods.

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J. THIE

Argonne National Laboratory
Lemont, Illinois

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Delayed Neutron Effects During Flux Tilt Transients

In the July issue of this Journal, two separate articles treat the problem of reactor flux during transients (*1*, *2*). Each of these articles directs some discussion to the effect of the presence of delayed neutrons on the spatial flux *shape* during a transient. We would like to add a few comments to this discussion. In general, the transient flux shape is not significantly affected by delayed neutrons if the reactor is small. However, in a reactor which has a dimension twenty times the migration length or more, delayed neutron effects on the transient flux shape begin to be significant. Such dimensions are not unheard of, and in particular, cores of roughly annular geometry such as PWR may have such dimensions around the circumference.

When an asymmetric instantaneous change in material properties is made, the flux shape changes part way toward its asymptotic form in a few prompt neutron lifetimes, but the remaining change takes place in times characteristics of the delay precursors. In this respect, the behavior of the shape is reminiscent of the behavior displayed by the fundamental mode when reactivity is inserted; that is, the reactor first experiences a prompt step change in power level, followed by a slower variation governed by the delayed neutrons. The influence of the delayed neutrons on the transient power shape depends upon how large a part of the flux tilt takes place promptly.

This effect may be seen most easily in a simple model, an initially uniform bare core with one energy group and one group of delayed precursors. At time zero, let a nonuniform perturbation, $\delta\Sigma_a(r)$, be made in the absorption cross section in such a way that the reactor remains critical. After this perturbation, the governing equations are

$$\left[\nabla^2 + B_{m0}^2 + B_{m1}^2 - \beta \frac{\nu\Sigma_{f0}}{D_0}\right]\phi + \frac{\lambda C}{D_0} = \frac{1}{D_0\, v}\frac{\partial\phi}{\partial t}, \qquad (1)$$

$$\frac{\partial C}{\partial t} = \beta\nu\Sigma_{f0}\,\phi - \lambda C, \qquad (2)$$

where

$$B_{m0}^2 = \frac{\nu\Sigma_{f0} - \Sigma_{a0}}{D_0}, \qquad B_{m1}^2 = -\frac{\delta\Sigma_a(r)}{D_0}, \qquad (3)$$

and the subscript zero indicates unperturbed quantities.

We may examine the transient behavior of the flux shape by a first-order perturbation theory and a modal expansion. If this is done, the equations governing the expansion coefficients are

$$\left[-B_{gk}^2 + B_{m0}^2 - \beta\frac{\nu\Sigma_{f0}}{D_0}\right]a_k + \frac{\lambda}{D_0}c_k = \frac{1}{D_0 v}\frac{d}{dt}a_k - \int \psi_k\, B_{m1}^2\,\phi_0\,, \qquad (4)$$

$$(d/dt)c_k = \beta\,\nu\Sigma_{f0}a_k - \lambda\, c_k\,, \qquad (5)$$

where a_k is the coefficient of the kth mode in the expansion of the flux perturbation, and c_k is the coefficient of the kth mode in the expansion of the precursor density perturbation. a_0 and c_0 vanish, since we have chosen the disturbance such that the reactor remains critical.

These equations may be readily solved. Figure 1 shows the time dependence of a typical coefficient, a_k, for various values of the parameter $\Delta B_g^2 = B_{gk}^2 - B_{g0}^2$. These curves show that the time dependence of the flux shape consists of a "prompt" jump plus a slow approach to the asymptote with time constant the order of a delayed neutron lifetime. As the buckling difference becomes smaller, the "prompt" jump becomes a smaller part of the whole.

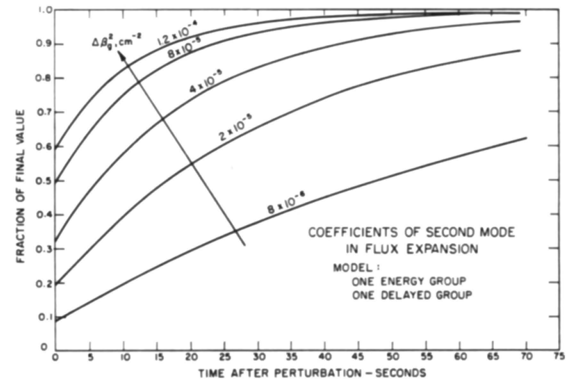

FIG. 1. Time behavior of expansion coefficient.

Solving Eqs. (4) and (5) analytically, one may find that the fractional height of the initial jump is

$$\frac{a_k(0+\epsilon)}{a_k(\infty)} \approx \frac{1}{1 + (\beta k_\infty / M^2 \Delta B_{gk}^2)}. \qquad (6)$$

For a one-dimensional reactor of length L, $\Delta B_{gk}^2 = k(k+2)(\pi^2/L^2)$.

Equation (6) shows that for small graphite-moderated cores of the type examined in reference *2*, in which both M^2 and ΔB_g^2 are large, no substantial effect on the shape is produced by the delayed neutrons. However, for hydrogen-moderated cores with at least one large dimension, the effect of delayed neutrons may be quite significant. For example, if the migration area is about 100 cm^2, the core length is 500 cm, and $k_\infty \approx 1$; then

$$(\beta k_\infty / M^2 \Delta B_{g1}^2) \approx 0.63,$$

and in this case, only about two-thirds of the asymptotic first-harmonic component manifests itself during the prompt jump.

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S. KAPLAN AND S. G. MARGOLIS

Westinghouse Electric Corporation
Pittsburgh, Pennsylvania

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Analysis of Neutron Flux Data for Accurate Determination of Relaxation Length*

Experiments with exponential piles usually require the measurement of a relaxation length to obtain the essential information about moderators, fuels, or lattices. The relaxa-

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