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Letters to the Editor

A Method for Interpolation in Moderator to Fuel Ratio Dependent Bucklings

It has become a tradition in both reactor theory and experiment to present calculated or measured bucklings as a function of the moderator to fuel ratio for fixed values of other lattice conditions. There is an optimal ratio which maximizes buckling, and usually lattices not too far from optimum are the ones of practical interest. Since bucklings are only known after much effort for discrete conditions, it may be useful to have a simple, yet accurate, method of interpolation.

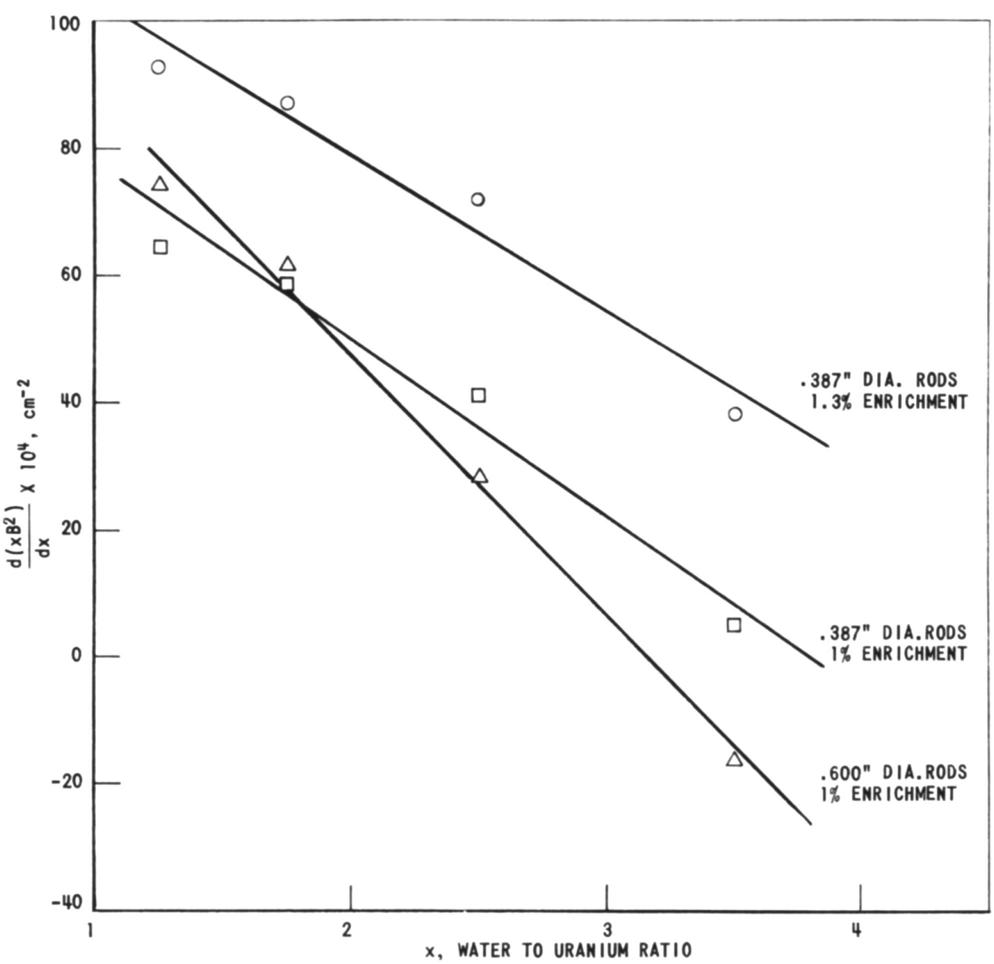

FIG. 1. Testing for the parabolic nature of xB^2 by looking for linearity in its derivative. The bucklings used are from theoretical calculations by Kouts *et al.* (*3*).

If x is the moderator to fuel ratio, then an obvious procedure for obtaining $B^2\,(x)$ from $n + 1$ known values, $B^2\,(x_i)$, is Lagrangian interpolation [*1*];

$$B^2(x) = \sum_{i=0}^{n} L_i B^2(x_i) \tag{1}$$

where

$$L_0 = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} \cdots \frac{x - x_n}{x_0 - x_n}$$
$$L_1 = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} \cdots \frac{x - x_n}{x_1 - x_n}, \text{ etc.} \tag{2}$$

This method essentially fits a polynomial of order n to $n + 1$ points, and is quite simple and rapid if the calculations are properly systematized [2]. However, the precision of the interpolated result at x can be improved upon if the function being interpolated is indeed a polynomial, preferably of low order. It will now be shown that $xB^2(x)$ is approximately parabolic in x and hence, capable of more accurate interpolation than is $B^2(x)$.

Assume that resonance escape, thermal utilization, and thermal migration area are the only lattice constants whose changes with x are typically the most significant and can be approximated as

$$p = \exp(-c_1/x) \tag{3}$$

$$f = 1/(1 + c_1 x) \tag{4}$$

$$L^2 = L_0^2(1 - f) = L_0^2[1 - (1 + c_2 x)^{-1}] \tag{5}$$

then in simple one group theory

$$B^2 = \frac{\epsilon\eta p f - 1}{L^2 + \tau} \tag{6}$$

$$\frac{dB^2}{dx} = \frac{k_\infty c_1}{M^2} \cdot \left(\frac{1}{x^2} - \frac{1}{x_m^2}\right) \tag{7}$$

The maximum buckling is at $x = x_m$, where

$$\frac{1}{x_m^2} = \left(\frac{L_0^2 B^2}{k_\infty} + \frac{1}{f}\right)\frac{c_2 f^2}{c_1} \tag{8}$$

Since c_1 and c_2 typically are weakly dependent on x, and since other lattice constants are slowly varying compared to $x^{-2} - x_m^{-2}$ near x_m, an integration of Eq. (7) yields

$$B^2 = -A_1 x + A_2 - (A_3/x) \tag{9}$$

The A_i are approximately constant. Therefore, xB^2 is approximately parabolic in x near x_m.

Figure 1 shows an example of the extent to which xB^2 is parabolic by testing for linearity in $d(xB^2)/dx$ from rigorously calculated bucklings [3]. This almost linear behavior in the derivative has been found among many other calculated and experimental bucklings.

The above supposition that

$$B^2(x) = \frac{1}{x} \sum_{i=0}^{n} L_i\, x_i\, B^2(x_i) \tag{10}$$

is more accurate than Eq. (1) by virtue of the parabolic nature of $xB^2(x)$ has indeed been found to be the case in practice. Table I compares Eqs. (1) and (10) as interpolation techniques for obtaining B^2 (1.5) and B^2 (3) from the calculated bucklings, B^2 (1), B^2 (2), and B^2 (4). It is seen that Eq. (10) is a few times more accurate than Eq. (1). Also, it seems to give surprisingly good interpolated values when compared with the calculated bucklings at $x = 1.5$ and 3, in spite of the factor of 2 interval size used for interpolation.

It is believed that Eq. (10) can be useful in connection with both theory and experiment. For the former, parameter studies made in reactor design and evaluation could conceivably involve fewer buckling calculations without loss of appreciable precision. The most precise interpolation between experimental points should be based on theoretical calculations of buckling. Lacking the latter, Eq. (10) should prove useful. This is especially the case for high precision experimental points, which might otherwise have a "draftman's eye" type curve drawn through them.

TABLE I

COMPARISON OF INTERPOLATION TECHNIQUES FOR BUCKLINGS

Moderator to fuel ratio, x	Calculated B^2 in μB, [3]	Errors of interpolated bucklings in μB: Ordinary linear interpolation using x_i = 1, 2	Ordinary linear interpolation using x_i = 2, 4	Interpolation by Eq. (1) using x_i = 1, 2, 4	Interpolation by Eq. (10) using x_i = 1, 2, 4
1	2671[a]				
1.5	4791[a]	−568		−305	134
2	5774[a]				
3	6256[a]		−541	513	−190
4	5655[a]				
1	1022[b]				
1.5	2830[b]	−522		−281	121
2	3593[b]				
3	3757[b]		−488	477	−166
4	2945[b]				
1	1553[c]				
1.5	3523[c]	−655		−361	128
2	4183[c]				
3	3729[c]		−440	734	−49
4	2394[c]				

[a] This set of bucklings used 0.387 in. diameter, 1.3% enriched uranium rods.

[b] This set of bucklings used 0.387 in. diameter, 1.0% enriched uranium rods.

[c] This set of bucklings used 0.600 in. diameter, 1.0% enriched uranium rods.

REFERENCES

1. F. B. HILDEBRAND. "Introduction to Numerical Analysis," P. 60. McGraw-Hill, New York, 1956.
2. L. COMRIE, "Chamber's 6-Figure Mathematical Tables," Vol. 2, p. XXX. Van Nostrand Company, Princeton, New Jersey, 1949.
3. H. KOUTS *et al.* Physics of slightly enriched, normal water. lattices (theory and experiment). *Proc. Intern. Conf. Peaceful Uses Atomic Energy, Geneva, 1958,* Paper P/1841.

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Delayed Neutron Effects During Flux Tilt Transients

In the July issue of this Journal, two separate articles treat the problem of reactor flux during transients (*1, 2*). Each of these articles directs some discussion to the effect of the presence of delayed neutrons on the spatial flux