## LETTERS TO THE EDITORS

# Space and Energy Separability of Thermal* Flux in a Diffusing Medium

In a letter to the editor of this journal (*1*), DeCoulon and Zweifel have raised questions regarding the validity of assumptions of separability in thermal neutron flux calculations. This communication will treat one of the situations discussed in the aforementioned letter, namely, the steady-state diffusion of neutrons from a thermal plane source in an infinite medium. All work reported here deals exclusively with light water moderators.

Results of a series of calculations using the SLOP-1 code[1] indicate that the thermal flux does fall off exponentially beyond a transient region, and that an asymptotic spectrum is attained. Of course, any treatment of the behavior of thermal neutrons in water remains inconclusive as long as the nature of the scattering kernel is in doubt. The problems described in the following were solved for a 36 group transfer matrix,[2] derived from a unit mass Wigner Wilkins kernel (*2*). Anisotropic transfer is included in the matrix. Qualitative conclusions drawn from this admittedly oversimplified model may, nevertheless, be valid in more realistic situations.

Three cases were studied. These differed slightly in geometric details, but were very similar in their essential features. In brief, a plane source of thermal neutrons was placed in a large volume of water, poisoned with a $1/v$ absorber. The poison concentration varied from problem to problem. Maxwell average poison cross sections for each case are listed in Table I.

Source neutrons were assumed to be slowed down into the thermal range through collisions with free protons in thermal motion. Within the thermal range itself, upscattering was neglected above 0.648 ev. Multigroup flux shapes were obtained in a $P_1$ approximation.

Figure 1 is a plot of number density as a function of distance from the source. The boundary of the source region is at the origin, and the region represented in the graph is source-free. It will be seen from Fig. 1 that the width of the transient region, measured in diffusion lengths, increases with the poison density. The behavior of the spectrum, in the transient region is illustrated in Fig. 2, which shows data taken from case 2. The three curves represent the energy spectrum at 0, 2.68, and 8.76 diffusion lengths, respectively. As might be expected, these curves differ most at the top of the thermal energy band. The flux near the thermal cut point is attenuated rapidly as the neutrons diffuse away from the source. This phenomenon appears to determine the width of the transient region, as long as one deals with a slowing-down source. If the source had a Maxwellian spectrum, the transient behavior would be quite different and the transient region would probably be smaller.

Diffusion length studies, as well as the other calculations for which SLOP-1 has been used, will be repeated with other kernels, which should be available shortly.

TABLE I

UNHARDENED POISON CROSS SECTIONS

| Case No. | Maxwell average $\Sigma_p$ in inverse cm ($T$ = 21°C) |
|---|---|
| 1 | 0.0195 |
| 2 | 0.0978 |
| 3 | 0.1950 |

* Work performed under the auspices of the U.S. Atomic Energy Commission.

[1] SLOP-1 is an IBM 704 multigroup code with upscattering. It will be described in detail in a forthcoming Bettis report.

[2] This matrix is generated by an auxiliary IBM 704 code called ECESS, developed by W. W. Clendenin.

## REFERENCES

1. G. DECOULON AND P. F. ZWEIFEL, *Nuclear Sci. and Eng.*, **5**, (1959).
2. E. P. WIGNER AND J. E. WILKINS, JR., "Effect of the Temperature of the Moderator on the Velocity Distribution of Neutrons with Numerical Calculations for H as Moderator," AECD 2275 (September, 1944).

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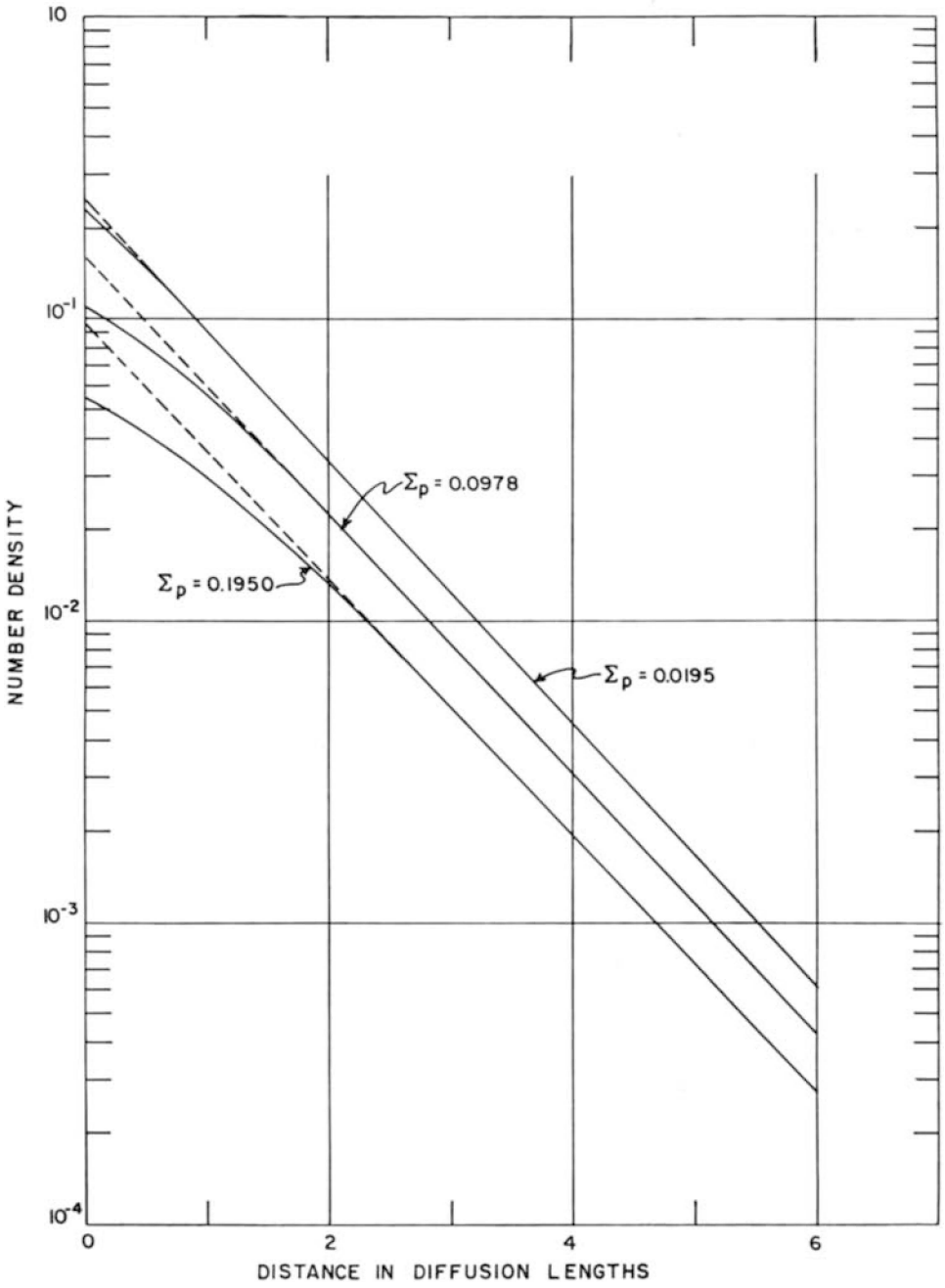


FIG. 1. Number density as a function of position for three different absorptions. Dotted lines represent exponentials tangent to the number density curves. Source lies to the left of the origin.

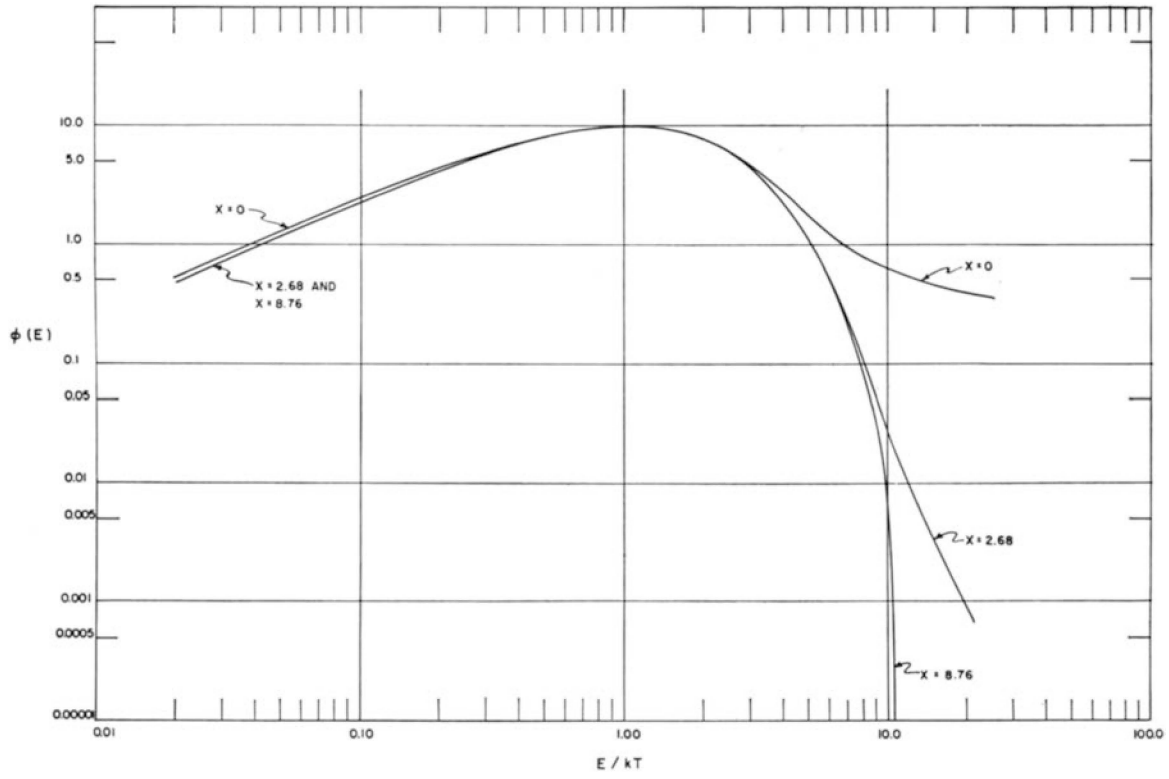


FIG. 2. Thermal flux, as a function of energy, at three different spatial points

## Validity of the $B_l$ Method for Fuel Cycle Analysis

The method for burnup analysis of fixed fuel thermal reactors (*1*) makes use of two simplifications, (a) separability of flux in space and time, and (b) representation of flux by the bare-equivalent flux shape. The purpose of this letter is to establish the conditions for validity of these approximations and to extend the analysis by the use of additive corrections to the burnup functions. For large reactors the correction is shown to be unimportant.

The first-order deviation from constant flux shape $\phi(\mathbf{r})$ may be represented by a single harmonic term added to the fundamental, so that

$$\Phi(\mathbf{r}, t) = \phi_F(\mathbf{r})c(t) + \phi_H(\mathbf{r})h(t). \tag{1}$$

It is shown in (*2*) that the $B_l$ theory is corrected to first order for changes in flux shape by using $B_l(x) - yH_l(x)$ wherever $B_l(x)$ occurs, with

$$H_l(x) = \frac{1}{V}\int_V \phi_F{}^l \phi_H \, e^{-\phi_F x}\, dV$$

$$x = \sigma \int_0^t c(t')\, dt'$$

$$y = \sigma \int_0^t h(t')\, dt'.$$

The magnitude of $h(t)$ may be estimated by using the constant power condition together with two group diffusion theory. For a typical enriched reactor, it is found that $y$ is proportional to $x^2$ and its magnitude is less than $0.1x^2$. Since $H_l(x) < B_l(x)$, the correction for harmonics due to nonseparability of flux is negligible for moderate burnup, i.e., $x < 1$.

The departure from bare-equivalent flux shape near the core-reflector interface is considered by taking the two group fluxes and adjoints in the general form

$$\Phi_i = c(t)[X(\mathbf{r}) + b_i Y(\mathbf{r})] \tag{2}$$

the $b_i$ being constants. The functions $X(\mathbf{r})$ and $Y(\mathbf{r})$ for the infinite slab are cos $r$ and cosh $ar$, respectively, where $r = \mu z$ and $a = \nu/\mu$. Use of the flux expressions (2) in a treatment parallel to that in Ref. (*1*) leads to equations for the quantities of interest during burnup. In addition to the regular burnup functions

$$B_l(x) = \frac{1}{V}\int_V X^l(\mathbf{r})\, e^{-X(\mathbf{r})x}\, dV$$

the equations involve new burnup functions containing $Y(\mathbf{r})$:

$$R_l(x) = \frac{1}{V}\int_V X^l(\mathbf{r})Y(\mathbf{r})e^{-X(\mathbf{r})x}\, dV$$

$$S_l(x) = \frac{1}{V}\int_V X^l(\mathbf{r})Y^2(\mathbf{r})e^{-X(\mathbf{r})x}\, dV, \quad \text{etc.}$$