LETTERS TO THE EDITORS

Comments on Bell's Approximation to the Resonance Integral

Bell (*1*) has recently suggested that in a dense lattice the quantity $S/(4\ V_0) = S_0$ be replaced by

$$\tau_0 = S_0[1 + (S_0V_0/\Sigma_1V_1)]^{-1}$$

in the calculation of resonance integrals. Here, S is the surface area of a lump, V_0 its volume, Σ_1 the moderator scattering cross section, and V_1 the volume of moderator associated with each lump. This suggestion has considerable practical utility and by restricting ourselves to slabs we can make a quick comparison with the Dancoff-Ginsburg correction and thus get an appreciation of the errors involved.

For slab lattices with a spacing d between slabs of fuel, Bell's formula yields for the ratio of effective surface area to actual surface area

$$\frac{S_{\text{eff}}}{S_{\text{Bell}}} = \frac{2\Sigma_1 d}{1 + 2\Sigma_1 d} \tag{1}$$

The Dancoff-Ginsberg correction factor (*2*) is:

$$\left(\frac{S_{\text{eff}}}{S}\right)_{\text{D-G}} = [1 - 2E_3(\Sigma_1 d)] \tag{2}$$

If we expand the correction factors about $\Sigma_1 d = 0$, we find

$$\left(\frac{S_{\text{eff}}}{S}\right)_{\text{D-G}} - \left(\frac{S_{\text{eff}}}{S}\right)_{\text{Bell}} = + (\Sigma_1 d)^2\left\{\ln \Sigma_1 d - \frac{5}{2} + \gamma\right\} - (\Sigma_1 d)^3\frac{25}{3} + \text{higher order terms} \tag{3}$$

where $\gamma = 0.5772$.

For dilute lattices, $E_3(\Sigma_1 d) \sim e^{-\Sigma_1}/\Sigma_1 d$, and Bell's approximation is considerably in error, as expected, but the error is in a term which is itself small. At $\Sigma_1 d = \frac{1}{2}$, however, $(S_{\text{eff}}/S)_{\text{D-G}} = 0.5568$ and $(S_{\text{eff}}/S)_{\text{Bell}} = 0.5$, which is in error by 10 per cent. This leads to an error of about 1 per cent in the resonance escape probability.

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Received March 30, 1959

Solution Of The P_3 Equations In One and Two Dimensions*

A numerical procedure for solving the spherical harmonics equations has been discussed in an earlier paper (*1*) describing the FLIP code. The method used in FLIP to treat one-group, one-dimensional slab problems, has now been applied to P_3 problems in other geometries. Extensions of the basic technique are incorporated in (1) the IBM-704 CLIP[1] code, which solves, numerically, the one-group, one-dimensional P_3 equations in cylindrical geometry and, (2) the IBM-704 TRIP[1] code, a one-group, two-dimensional P_3 code in Cartesian coordinates.

Both codes include all anisotropic scattering components permitted in a P_3 approximation. Both are designed exclusively for cell problems.

A typical CLIP problem with 100 mesh points requires about 3 min of machine time, including the time necessary to process input and output data. Typical times for TRIP, on the other hand, vary over a wide range depending, roughly, on the magnitude of the quantity h^2/L^2. Here h is the mesh width, and L the diffusion length. If the mesh width, measured in diffusion lengths is small, convergence will be slow. A problem with 2500 mesh points[2] may consume anywhere from $1\frac{1}{2}$ to 3 hr of machine time. It should be noted however, that experience with TRIP has suggested a modification which might decrease this time substantially.

The basic equations in CLIP and TRIP are derived from the conventional P_3 equations by eliminating spherical harmonics of odd order. Coupled second-order differential equations result. Unfortunately, the coupling terms contain derivatives of the various flux components, and it appears to be impossible to eliminate these terms. Nevertheless, the equations may be treated by technique developed for the solution of the diffusion equation. Thus, the major sections of TRIP were taken bodily from PDQ3, a four-group diffusion code (*2*).[3] The powerful overrelaxation process used in PDQ3 is just as effective in TRIP.

The two-dimensional P_3 code has been used to study flux peaking in the neighborhood of cruciform water channels. Results have been compared with those obtained with the Monte Carlo TUT (*3*) code. This comparison suggests that the P_3 fluxes are too high in the water and in those parts of

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* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] CLIP was programed by B. L. Anderson and P. H. Jarvis, TRIP by J. Dorsey, J. Mandel, and H. Henderson.

[2] This is the maximum number of interior mesh points.

[3] PDQ3 is a revised version of the earlier PDQ2 (see Ref. *2*).

the fuel adjacent to it. However, present Monte Carlo methods are not designed to provide detailed flux distributions. Even after 80,000 case histories statistical fluctuations in the TUT averages preclude any definitive evaluation of the accuracy of P_3 for this problem. With the advent of faster electronic computers, more detailed and more accurate Monte Carlo results may soon be available for comparison.

REFERENCES

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Received July 20, 1959

Fission Counter Sample Disk Holder and Slide

One method of analyzing uranium compounds to determine the percentage of the uranium isotope U^{235} is to bombard a thin film of the compound with neutrons and measure the ionization resulting from the fission of the U^{235} isotopes present in the film. This method of analysis is best adapted to nonvolatile compounds and is a common method of assay analysis employed at U. S. gaseous diffusion plants. Special counters have been designed and constructed for this purpose at each of these plants. Basically these counters consist of a radium-beryllium radiation source, four ionization chambers housed in a lead shielding assembly, and the necessary electronic equipment for the measurement and indication of ionization. After chemical preparation, a sample of the material to be analyzed is electroplated onto a thin smooth nickel disk. The disk is mounted on a holding plate which is attached to a slide assembly. When inserted into the radiation assembly, the slide positions the mounting plate and sample disk for radiation and ionization measurement. In two of the special counters the plated sample-disk is position on a round mounting plate over which an annular retaining frame is mounted to secure the disk. The frame is held to the mounting plate by spring loaded retaining balls. The disk is then inserted into the counter on a slide device to which the mounting plate is attached. Inside the radiation assembly the sample disk is positioned opposite a collector plate which serves as the sensing element for measuring the ionization. Before a sample is irradiated or "counted," the area between and adjacent to the sample disk and collector plate must be purged with nitrogen to provide a standard atmosphere for each analysis.

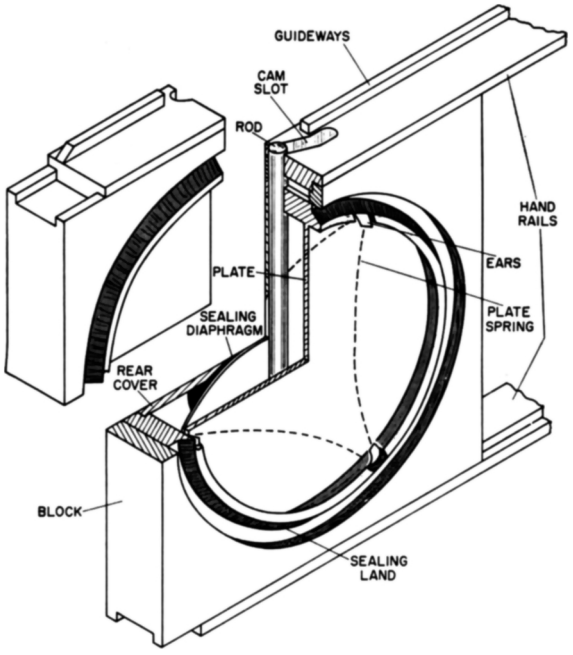

FIG. 1. Fission counter slide holder

The fission counter constructed for use by Goodyear Atomic Corporation is designed to utilize a hemispherical ionization chamber with which the sample disk mounting plate can form a gas-tight seal. This increases the accuracy of the electronic measurement of ionization and provides a confined ionization atmosphere which can be purged and retained for the entire count of the sample. It also provides for reduced purge gas consumption and permits the economic use of a more favorable purge gas. Design modifications provide for a more efficient method of mounting the disk on the sample holder slide assembly. It was necessary to design the sample disk holder slide so that it would vertically position the disk inside the radiation assembly, and then move the disk and mounting plate forward horizontally to engage the rubber O-ring at the mouth of the chamber to form a gas-tight seal. The mechanism designed to provide this motion and a rapid and positive method of mounting the sample disk is shown in Fig. 1.

To place the sample disk on the mounting plate a slight finger pressure is exerted on the portion of the sealing diaphragm exposed through the hole in the rear cover of the mounting plate. This pressure is exerted directly on the center of the triangular-shaped plate spring causing the ears to open at the face of the plate. The disk is placed on the plate face, film side out, and the finger pressure on the spring is then released. The spring contracts, causing the ears to close on the disk, clamping it against the face of the mounting plate.

The slide is then inserted into the radiation assembly and is guided into position by the tongue-and-groove guideways. When the block reaches the bottom of the radiation assembly, the continued downward motion of the handle rails causes the rod to be moved forward by the cam slot. The horizontal movement of the rod in following the cam slot