LETTERS TO THE EDITORS

Comments on Bell's Approximation to the Resonance Integral

Bell (*1*) has recently suggested that in a dense lattice the quantity $S/(4\ V_0) = S_0$ be replaced by

$$\tau_0 = S_0[1 + (S_0V_0/\Sigma_1V_1)]^{-1}$$

in the calculation of resonance integrals. Here, S is the surface area of a lump, V_0 its volume, Σ_1 the moderator scattering cross section, and V_1 the volume of moderator associated with each lump. This suggestion has considerable practical utility and by restricting ourselves to slabs we can make a quick comparison with the Dancoff-Ginsburg correction and thus get an appreciation of the errors involved.

For slab lattices with a spacing d between slabs of fuel, Bell's formula yields for the ratio of effective surface area to actual surface area

$$\frac{S_{\text{eff}}}{S_{\text{Bell}}} = \frac{2\Sigma_1 d}{1 + 2\Sigma_1 d} \tag{1}$$

The Dancoff-Ginsberg correction factor (*2*) is:

$$\left(\frac{S_{\text{eff}}}{S}\right)_{\text{D-G}} = [1 - 2E_3(\Sigma_1 d)] \tag{2}$$

If we expand the correction factors about $\Sigma_1 d = 0$, we find

$$\left(\frac{S_{\text{eff}}}{S}\right)_{\text{D-G}} - \left(\frac{S_{\text{eff}}}{S}\right)_{\text{Bell}} = + (\Sigma_1 d)^2\left\{\ln \Sigma_1 d - \frac{5}{2} + \gamma\right\} - (\Sigma_1 d)^3\frac{25}{3} + \text{higher order terms} \tag{3}$$

where $\gamma = 0.5772$.

For dilute lattices, $E_3(\Sigma_1 d) \sim e^{-\Sigma_1}/\Sigma_1 d$, and Bell's approximation is considerably in error, as expected, but the error is in a term which is itself small. At $\Sigma_1 d = \frac{1}{2}$, however, $(S_{\text{eff}}/S)_{\text{D-G}} = 0.5568$ and $(S_{\text{eff}}/S)_{\text{Bell}} = 0.5$, which is in error by 10 per cent. This leads to an error of about 1 per cent in the resonance escape probability.

REFERENCES

1. G. I. BELL, "A Simple Treatment for Effective Resonance Absorption Cross Sections in Dense Lattices," *Nuclear Sci. and Eng.* **5**(2), 138 (1959).
2. S. M. DANCOFF AND M. GINSBURG, CP-2157 (1944).

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Received March 30, 1959

Solution Of The P_3 Equations In One and Two Dimensions*

A numerical procedure for solving the spherical harmonics equations has been discussed in an earlier paper (*1*) describing the FLIP code. The method used in FLIP to treat one-group, one-dimensional slab problems, has now been applied to P_3 problems in other geometries. Extensions of the basic technique are incorporated in (1) the IBM-704 CLIP[1] code, which solves, numerically, the one-group, one-dimensional P_3 equations in cylindrical geometry and, (2) the IBM-704 TRIP[1] code, a one-group, two-dimensional P_3 code in Cartesian coordinates.

Both codes include all anisotropic scattering components permitted in a P_3 approximation. Both are designed exclusively for cell problems.

A typical CLIP problem with 100 mesh points requires about 3 min of machine time, including the time necessary to process input and output data. Typical times for TRIP, on the other hand, vary over a wide range depending, roughly, on the magnitude of the quantity h^2/L^2. Here h is the mesh width, and L the diffusion length. If the mesh width, measured in diffusion lengths is small, convergence will be slow. A problem with 2500 mesh points[2] may consume anywhere from $1\frac{1}{2}$ to 3 hr of machine time. It should be noted however, that experience with TRIP has suggested a modification which might decrease this time substantially.

The basic equations in CLIP and TRIP are derived from the conventional P_3 equations by eliminating spherical harmonics of odd order. Coupled second-order differential equations result. Unfortunately, the coupling terms contain derivatives of the various flux components, and it appears to be impossible to eliminate these terms. Nevertheless, the equations may be treated by technique developed for the solution of the diffusion equation. Thus, the major sections of TRIP were taken bodily from PDQ3, a four-group diffusion code (*2*).[3] The powerful overrelaxation process used in PDQ3 is just as effective in TRIP.

The two-dimensional P_3 code has been used to study flux peaking in the neighborhood of cruciform water channels. Results have been compared with those obtained with the Monte Carlo TUT (*3*) code. This comparison suggests that the P_3 fluxes are too high in the water and in those parts of

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* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] CLIP was programed by B. L. Anderson and P. H. Jarvis, TRIP by J. Dorsey, J. Mandel, and H. Henderson.

[2] This is the maximum number of interior mesh points.

[3] PDQ3 is a revised version of the earlier PDQ2 (see Ref. *2*).