TABLE I

DECAY OF THERMAL NEUTRON DENSITY IN A 20 BY 20⅛ BY 20⅛ IN. BERYLLIUM ASSEMBLY AS A FUNCTION OF TEMPERATURE[a]

T, Temperature °C	T, Temperature °K	λ (10^3 sec^{-1})	λ_{tr} (cm)
		(× 10^3	
−46	227	1.482	1.58
27	300	1.560	1.47
50	323	1.593	1.45
100	373	1.665	1.43
146	419	1.720	1.40
220	493	1.815	1.38
233	506	1.850	1.39
292	565	1.922	1.37
335	608	1.992	1.38
511	784	2.220	1.38

[a] These reported temperatures were determined to ±5° and the decay constants to ±2 per cent; ρ, density of beryllium, = 1.85 gm/cm^3, B^2, buckling, = 1.05 × 10^{-2} cm^{-2}.

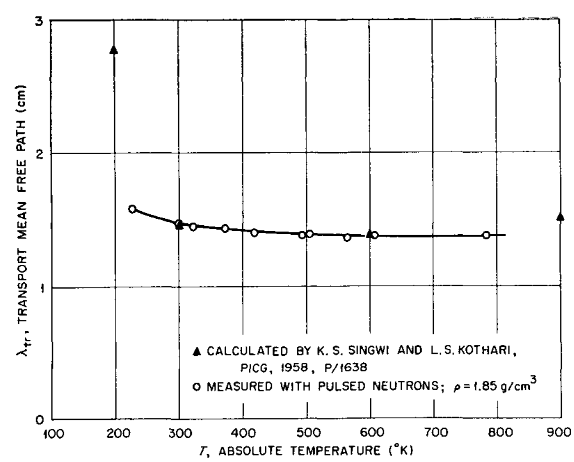

FIG. 1. Transport mean free path of thermal neutrons in beryllium at various temperatures.

and the transport mean free path was assumed to be

$$\lambda = \lambda_a + (\tfrac{1}{3})\lambda_{tr} v B^2(1 - CB^2) \qquad (1)$$

where $v = (8kT/\pi m)^{1/2}$ is the average neutron velocity; k is the Boltzmann constant; m, the mass of the neutron; and T, the absolute temperature of the beryllium. For the determination of λ_{tr} from Eq. (1), $B^2(1 - CB^2)$ was taken as temperature-independent; this is justified in this case since $CB^2 \ll 1$ and the extrapolation length is only about 4 per cent of the linear dimensions of the beryllium assembly.

The measured decay constants and the corresponding values of the transport mean free path are given in Table I. Figure 1 shows the experimentally determined variation of λ_{tr} with temperature and also four values of λ_{tr} calculated by K. S. Singwi and L. S. Kothari (*1*) (corrected to a density of 1.85 gm/cm^3). It is seen from Fig. 1 that the agreement between the experiment and the calculations of Singwi and Kothari is good.

These measurements will be extended toward lower temperatures as soon as the necessary experimental equipment is completed. An investigation of the variation of the diffusion cooling constant C with temperature is also in progress now and will be described in a later paper.

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A New Approximate Treatment of the Energy-Dependent Boltzmann Equation

For a constant cross section, nonabsorbing medium, the one-dimensional Boltzmann equation can be written (*1*)

$$\mu\phi' + \phi = \frac{1}{2\pi}\int_0^u du' \int_{-1}^1 d\mu' \int_0^{2\pi} d\varphi'\phi(x, u'\, \mu')f(\mu_0, u - u') + S(x, u, \mu)$$

where distance is expressed in terms of the scattering length.

Let us consider a unit isotropic plane source,

$$S(x, u, \mu) = \delta(x)\delta(u)$$

If we now take the Laplace transform with respect to lethargy we get

$$\mu\bar{\phi}' + \bar{\phi} = \frac{1}{2\pi}\int_{-1}^1 d\mu' \int_0^{2\pi} d\varphi'\bar{\phi}(x, s, \mu')\bar{f}(s, \mu_0) + \delta(x)$$

By expanding $\bar{f}(\mu_0, s)$ in a Legendre series

$$\bar{f}(\mu_0, s) = \sum_{l=0}^{\infty} \frac{2l+1}{2}\bar{f}_l(s)P_l(\mu_0)$$

and approximating $\bar{\phi}(x, s, u)$ as[1]

$$\bar{\phi}(x, s, \mu) = \tfrac{1}{2}\bar{\phi}_0(x, s)P_0(\mu) + \tfrac{3}{2}\bar{\phi}_1(x, s)P_1(\mu)$$

one easily gets

[1] This is the P_1 approximation.

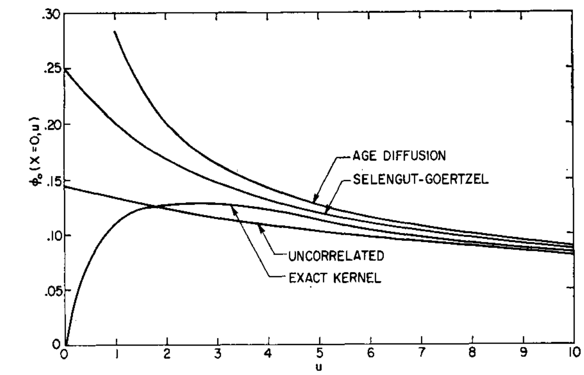

FIG. 1. P_1 solution of Boltzmann equation with various scattering kernels.

$$\phi_0(x, u) = \frac{1}{2}\mathfrak{L}^{-1}\sqrt{\frac{3(1-\bar{f}_1)}{(1-\bar{f}_0)}}\exp\left[-\sqrt{3(1-\bar{f}_1)(1-\bar{f}_0)}\,|x|\right]$$

For a purely hydrogenous medium

$$f(\mu, u) = e^{-u}\delta(\mu - e^{-u/2})$$

and we have

$$\bar{f}_0 = (s+1)^{-1}$$

$$\bar{f}_1 = (s + \tfrac{3}{2})^{-1}$$

With these values for $\bar{f}_0$ and $\bar{f}_1$ it is impossible to invert $\phi_0(x, s)$ analytically, and even when $x = 0$ the problem is not easily handled. It can, however, be inverted numerically, and the result for $\phi_0(0, u)$ is given in Fig. 1 ("exact kernel").

One approach at simplifying the problem comes about from expanding the $\bar{f}$'s in a series in s:

$$\bar{f}_0 = 1 - s + s^2 \cdots$$

$$\bar{f}_1 = \tfrac{2}{3}[1 - \tfrac{2}{3}s + \tfrac{4}{9}s^2 \cdots]$$

A simple approximation is to take two terms in the $\bar{f}_0$ expansion, and one term in the $\bar{f}_1$ expansion. This is the familiar age-diffusion approximation;[2] it gives

$$\bar{f}_0 = 1 - s \qquad \bar{f}_1 = \tfrac{2}{3}$$

$$\phi_0(x = 0, u) = \tfrac{1}{2}\mathfrak{L}^{-1}\sqrt{1/s} = 1/\sqrt{4\pi u}$$

This result is also plotted in Fig. 1. Although it has the proper asymptotic form for large u, it is quite poor for small values of u.

A somewhat better approximation comes about from treating $\bar{f}_0$ exactly, still taking only one term in the expansion of $\bar{f}_1$. This is the Selengut-Goertzel approximation,[3] and with it we get

$$\bar{f}_0 = (s+1)^{-1} \qquad \bar{f}_1 = \tfrac{2}{3}$$

$$\phi(x = 0, u) = \frac{1}{2}\mathfrak{L}^{-1}\sqrt{\frac{s+1}{s}}$$

$$= \frac{e^{-u/2}}{4}[I_0(u/2) + I_1(u/2)] \qquad u \neq 0$$

I_0 and I_1 are Bessel functions of purely imaginary argument. By looking at Fig. 1 we can see that this approximation is better than that of age diffusion but still quite poor for small values of u.

A different approach is to take a scattering kernel of the form

$$f(\mu, u) = 2\mu e^{-u} \qquad \mu > 0$$

$$= 0 \qquad \mu < 0$$

giving

$$\bar{f}_0 = (s+1)^{-1} \qquad \bar{f}_1 = \tfrac{2}{3}(s+1)^{-1}$$

This has the *exact* angular distribution, and the *exact* energy distribution; the two are, however, completely uncorrelated. That they are uncorrelated can be seen by noting that $\bar{f}_1/\bar{f}_0$ is not a function of s. That this is not true in the Selengut-Goertzel approximation means that there is some correlation there. These values of $\bar{f}_0$ and $\bar{f}_1$ give

$$\phi_0(x = 0, u) = \sqrt{\frac{3}{2}}\mathfrak{L}^{-1}\sqrt{\frac{s+\frac{1}{3}}{s}}$$

$$= \sqrt{\frac{3}{2}}e^{-u/6}[I_0(u/6) + I_1(u/6)] \qquad u \neq 0$$

By examining Fig. 1 one can see that this gives a still better representation of $\phi_0(x = 0, u)$ for small u, while retaining the correct asymptotic dependence.

Another check on these two latter techniques is to examine the value for the age given by each, since this is a measure of the values not only at $x = 0$, but throughout the entire range.

The P_1 approximation with the exact kernel gives the exact age, which for large values of u is

$$\frac{\bar{x}^2}{2} = u + \frac{2}{3} + O(e^{-u/2}) \qquad P_1\text{; exact kernel}$$

The other approximations give

$$\frac{\bar{x}^2}{2} = u + 2 \qquad P_1\text{; Selengut-Goertzel kernel}$$

$$\frac{\bar{x}^2}{2} = u + 0 + O(e^{-u/3}) \qquad P_1\text{; uncorrelated kernel}$$

In this case one can see that the error with uncorrelated kernel is only one-half as large as that with the Selengut-Goertzel kernel.

Thus both for the representation it gives to the lethargy-dependent flux at $x = 0$, and for the average spatial description it gives to the flux at large lethargies, the "uncorrelated" approximation appears to give better results than the Selengut-Goertzel approximation. The only advantage the Selengut-Goertzel procedure has is that for the case of nonconstant cross sections the scalar component of the flux can be obtained from

[2] It can be shown that the approximation to the integral

$$\int_0^u \phi_n(u')f_n(u-u')\,du'$$

obtained by taking r terms in a Taylor expansion of ϕ_n, is the same as taking r terms in a power series of $\bar{f}_n(s)$, as long as $(u - u') \geqq q_M$ (q_M is the maximum lethargy loss in one collision). Since age theory assumes $(u - u') \geqq q_M$ this is no restriction.

[3] Since $\bar{f}_1(s) = \frac{2}{3}$, $f_1(u) = \frac{2}{3}\delta(u)$; i.e., there is no energy loss by particles scattered in the P_1 mode. This is the same as expanding $\phi_1(u')$ in a Taylor series in u, and taking only the first term: $\phi_1(u)$.

$$-\frac{1}{\Sigma_H(u)}\nabla^2\phi_0 + \Sigma_H(u)\phi_0(u) = \int_0^u d\mu'\phi_0(u', x)e^{-(u-u')}\Sigma_H(u) + S_0;$$

(purely hydrogenous medium)

for an uncorrelated scattering kernel the corresponding equation is more complicated. However, even the above must usually be solved by a machine, and if a machine is to be employed, one might as well use the exact kernel rather than an approximate one. In those cases in which one can use the Selengut-Goertzel approximation analytically, the uncorrelated approximation appears preferable. (In some cases it might be best to use a combination of two-thirds uncorrelated scattering kernel and one-third Selengut-Goertzel scattering kernel, since this gives the correct asymptotic value for the age.)

The uncorrelated approximation can be applied easily to a more general medium containing a hydrogenous cross section Σ_H, an absorption cross section Σ_a, and an "infinitely heavy" scatterer with cross section Σ_0 and an average cosine theta of $\bar{\mu}$.

In this case one gets

$$\phi_0(x = 0, u) = \frac{m}{2} e^{-nu}[I_0(mu) + I_1(mu)] \sqrt{\frac{\Sigma_T - \bar{\mu}\Sigma_0}{\Sigma_H + \Sigma_a}} \qquad u \neq 0$$

$$m = \frac{1}{2}\left[1 - \frac{\frac{2}{3}\Sigma_H}{\Sigma_T - \bar{\mu}\Sigma_0} - \frac{\Sigma_a}{\Sigma_H + \Sigma_a}\right]$$

$$n = \frac{1}{2}\left[1 - \frac{\frac{2}{3}\Sigma_H}{\Sigma_T - \bar{\mu}\Sigma_0} + \frac{\Sigma_a}{\Sigma_H + \Sigma_a}\right]$$

$$\Sigma_T = \Sigma_H + \Sigma_a + \Sigma_0.$$

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The Age of Plutonium-Beryllium Neutrons in Light Water[1]

The disagreement between theoretically calculated and experimentally observed ages in hydrogenous media has led to a continuing interest in these measurements. Although many age experiments have been made using other than fission neutrons, it seems that a considerable amount of the work has been done with neutrons produced by polonium-beryllium sources which have several disadvantages (*1*) that can be avoided by the use of plutonium-beryllium sources.

Some measurements have been made in this Laboratory of the ages of plutonium-beryllium neutrons in light water to the indium (1.458 ev), rhodium (1.25 ev), and silver (5.18 ev) resonances.

[1] Work supported in part by the U. S. Atomic Energy Commission.

TABLE I

Neutron source	Neutron age (cm^2)		
	Ag^{109} (5.18 ev)	In^{115}(1.458 ev)	Rh^{103}(1.25 ev)
Pu-Be	49.3	52.8 ± 2.5	53.7 ± 2.5
Po-Be		57.3 ± 2.0	

The sources were suspended by means of stainless steel wires in a cylindrical tank about 4 ft in diameter and 4 ft in height. The cadmium-covered indium foils were placed in lucite holders which were suspended above the source by an aluminum rod. The foils were irradiated separately in order to avoid shadowing effects and each irradiation lasted for at least six hours. The activities were corrected in the usual manner for background, flux depression by the cadmium covers (*2*), and finite size of the source and foils (*3*). Activations by neutrons of energies higher than the resonance energy of interest were assumed to be negligible (*4*, *5*). The activation curves were plotted in the usual manner. The area under the exponential portion of the curve was obtained analytically and the remaining area evaluated by Simpson's rule. Preliminary results of these measurements are given in Table I.

The measurement of the age of the polonium-beryllium neutrons was made principally for the purpose of evaluating the dependability of the techniques used, and the result obtained appears to be in reasonable agreement with the average age found by other authors (*6*).

One of the disadvantages of the plutonium-beryllium source is its low specific activity and this, coupled with the low abundance of the 109-isotope, resulted in poor statistics for the case of silver. For this reason, no error is assigned to the age to the silver resonance (5.18 ev) as the value reported is considered to be only an indication of the true age.

A paper on these experiments is in preparation and will be submitted for publication at a later date.

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