by the crystal. The presence of neutrons of energy $< E_1$ has also been observed by Pattenden (7) during measurements with a Be crystal. The contamination of the diffracted beam by neutrons of energies different from the expected ones may be more pronounced at some angles. More careful measurements of the energy spectrum of the diffracted beam by a single crystal (when the incident beam is taken from a reactor) are desirable since this instrument has been widely used for cross-section measurements.

Thanks are due to Mr. L. S. Kothari for pointing out the inelastic scattering effects as a possible cause of the fluctuations and for help in the calculations. Thanks are also due to Messrs. Raghavendra Rao and K. Sri Ram for their help during measurements. Continued assistance of Messrs. J. N. Soni, Virendra Singh, and M. L. Barde is gratefully acknowledged. The author is thankful to Dr. R. Ramanna for his help throughout the course of this work.

Note Added in Proof

Calculations of the intensity of inelastically scattered neutrons were recently made at a few Bragg angles. The contributions of such neutrons at 17°45′ and 26° came out to be much larger than at neighboring angles, thus causing peaks at these angles. The details will be published shortly.

Similar fluctuations in the case of Be crystal have been explained by H. J. Hay, A.E.R.E. Harwell (Private communication, April 1959) as due to double Bragg-reflections. Spencer and Smith have also reported similar findings [*Bull. Am. Phys. Soc.* May 1, 1959] in Be and NaCl crystals.

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Heat Transfer to Water Flowing Parallel to Tube Bundles

A fuel element assembly consisting of cylindrical fuel rods cooled by water flowing parallel to the axis of the rods is one of the arrangements most frequently encountered by the reactor designer. For this situation, the usual procedure for calculation of nonboiling heat transfer coefficients in the fully turbulent region is to use a modified version of Colburn's (1) equation

$$\frac{hD_e}{k} = 0.023 \left(\frac{D_e G}{\mu}\right)^{0.8} \left(\frac{C_p \mu}{k}\right)^{1/3} \tag{1}$$

where h is the heat transfer coefficient, k the thermal conductivity of the fluid, C_p the specific heat of the fluid, μ the liquid viscosity, and D_e an equivalent diameter equal to four times the hydraulic radius. All the fluid properties are generally evaluated at the film temperature except C_p, which is taken at the bulk temperature.

The possibility that the effect of tube spacing is not adequately described by Eq. (1) has been considered by several investigators. Deissler and Taylor (2) studied the problem analytically and concluded that at a given Reynolds number, based on D_e, the more open lattices should provide higher heat transfer coefficients. Their results, however, were not presented in a form readily adaptable to engineering design.

Heat transfer coefficients on the shell side of unbaffled heat exchangers were experimentally investigated some years ago by Short (5). The lattice spacing effect he observed could not be described adequately by use of the equvalent diameter alone. However, his studies were confined to the flow transition region at Reynolds numbers between 10^3 and 10^4. More recently Wantland (6) investigated the heat transfer characteristics of two additional arrays in the transition region. The results of this study were also at variance with Eq. (1).

The primary concern of the reactor designer is with the fully turbulent region at Reynolds numbers above 2.5×10^4. Experimental studies of this region with water flowing outside of tube bundles have been carried out by Miller *et al.* (4) and Dingee *et al.* (3). For a lattice spacing where the ratio of the center to center distance between tubes, S, to the tube diameter, D, was 1.46, Miller *et al.* found the data could be described by an equation of the same form as Eq. (1), but with a different coefficient.

$$\frac{hD_e}{k} = C \left(\frac{D_e G}{\mu}\right)^{0.8} \left(\frac{C_p \mu}{k}\right)^{1/3} \tag{2}$$

The value of C was determined as 0.032. *Dinge et al.* (3) investigated several more closely spaced lattices and found that, while the data did not depart greatly from the results predicted by Eq. (1), the more open lattice spacings tended to give somewhat higher heat transfer coefficients. Both investigators took precautions to allow a sufficient downstream length of remove entrance effects.

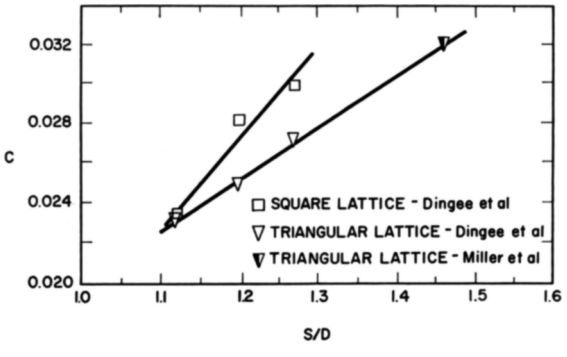

Fig. 1.

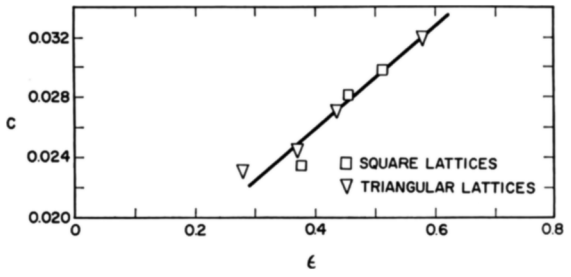

FIG. 2.

The method of least squares has been applied to the data available for the fully turbulent region, and the value of the coefficient, C, of Eq. (2) determined for each of the lattices investigated. As can be seen by the graphical representation of the results in Fig. 1, C appears to vary linearly with the S/D ratio. At the same S/D values, square pitch lattices, which are more open, give higher values of C than do the triangular pitch lattices. At Reynolds numbers between 2.5×10^4 and 10^6 we have for triangular pitch lattices, where S/D lies between 1.1 and 1.5

$$C = 0.026\ (S/D) - 0.006 \tag{3}$$

and for square pitch lattices, where S/D lies between 1.1 and 1.3

$$C = 0.042\ (S/D) - 0.024 \tag{4}$$

It is instructive to compare the results for square and triangular pitch lattices when plotted as a function of ϵ, the ratio of the water flow area to the total cross sectional area of an infinite lattice. As can be seen from Fig. 2, both lattice types yield essentially the same heat transfer coefficients at equivalent values of ϵ.

It should be noted that for almost all cases of interest, Eqs. (3) and (4) yield higher heat transfer coefficients than predicted by the Colburn equation. This should be an aid to the reactor designer since somewhat lower fluid velocities can now be used to obtain the high heat transfer coefficients desired.

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The Double Spherical Harmonic Method for Cylinders and Spheres

Several attempts to extend the double spherical harmonics method of Yvon (*1*) to cylindrical and spherical systems have appeared in the unpublished reactor technology literature. Different sets of differential equations for the same system have been suggested depending on the treatment of a product of singular functions which occurs in the analysis.

In this note we would like to point out that when one attempts to use the Yvon method for cylinders or spheres, one encounters the problem of finding the product $Y \cdot \delta$ of a Dirac δ function and a Heaviside step function, Y, that is, $Y(x) = 1$ for $x > 0$, $Y(x) = 0$ for $x < 0$. It is well known that even if one interprets these functions as distributions in the sense of L. Schwartz (*2*), the product $Y \cdot \delta$ is not defined. However, it is possible to make use of a product of distributions defined by H. Koenig (*3*) to obtain double spherical harmonics moment equations for the cylinder and sphere. The distribution product of $Y \cdot \delta$ defined by Koenig involves an arbitrary constant which must be determined by physical considerations. The same result, still involving an arbitrary constant, can be obtained without explicit use of distribution theory.

The one-velocity transport equation for a system with cylindrical symmetry may be written

$$\sin\theta\left[\cos\phi\,\frac{\partial}{\partial r}f(r,\theta,\phi) - \frac{\sin\phi}{r}\,\frac{\partial}{\partial\phi}f(r,\theta,\phi)\right] + \Sigma f(r,\theta,\phi)$$

$$= \int_0^{2\pi} d\phi' \int_0^{\pi} d\theta' \frac{\Sigma_S}{4\pi}(r,\cos\theta_0)\, f(r,\theta'\phi')\sin\theta' + S'(r,\theta,\phi)$$

We expand the scattering kernel in ordinary spherical harmonics and for simplicity keep only the first term corresponding to isotropic scattering. To expand the flux in the double P_1 approximation we start with the usual first two spherical harmonics required for a symmetrical solution, i.e., 1 and $\sin\theta\cos\phi$, and construct the corresponding nonorthogonal set of "double spherical harmonics"

$$F_1 = (2\pi)^{-1/2}A, \qquad F_2 = (2\pi)^{-1/2}B,$$

$$F_3 = \sin\theta\cos\phi\, A, \qquad F_4 = \sin\theta\cos\phi B$$

where

$$A = 1 \text{ for } -\pi/2 < \phi < \pi/2, \qquad 0 \text{ otherwise}$$

$$B = 1 \text{ for } \pi/2 < \phi < 3\pi/2, \qquad 0 \text{ otherwise.}$$

We can write A and B using Heaviside step functions as

$$A = Y_{-\pi/2} - Y_{\pi/2}$$

$$B = Y_{\pi/2} - Y_{3\pi/2}$$

A corresponding orthogonal set of functions spanning the same space is found by the Gramm-Schmidt process to be

$$F_1, F_2,$$

$$F_3 = (6/\pi)^{1/2}A(\sin\theta\cos\phi - \tfrac{1}{2})$$

$$F_4 = (6/\pi)^{1/2}B(\sin\theta\cos\phi + \tfrac{1}{2}).$$