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LETTERS TO THE EDITORS

Reactor Similitude

The art of nuclear engineering has not yet advanced to the point where analysis will successfully predict all phases of a nuclear design. Experiments are necessary to determine the completeness of the theory. A "cut and try" experimental method of design incurs a prohibitive cost and even the expense involved in the occasional mock-up of full scale systems has suggested the use of compromising models to predict performance. For example, in the field of hydrodynamics, giant flow systems are reproduced by a small scale replica, in which an attempt is made to duplicate the Reynolds, Froude, Mach, and Weber numbers and the pressure coefficient. Thus, it is hoped that two systems which are theoretically equivalent will also be experimentally equivalent. It is the author's purpose to review similar dimensionless parameters in the field of nuclear reactors that may be used for the simplification of nuclear experiments.

One must generally choose a specific interest since it is difficult to represent all phases of a phenomenon in a scaled nuclear model. For example, it may be possible to reproduce accurately over-all criticality with errors in local phenomena. Since the experimental models will at best be similar and not exact, it is only feasible to consider major effects. The parameters chosen for discussion are neutron leakage, thermal utilization, control, disadvantage factors, and then miscellaneous topics.

It may often be easier and cheaper to perform experiments on a model of reduced physical size. To maintain the leakage characteristics the model must conserve the dimensionless quantity $B_f^2\tau + B_t^2L^2$, where B_f^2 and B_t^2 are the fast and thermal bucklings, respectively, τ is the neutron age, and L is the thermal neutron diffusion length. For reflected reactors the second term is usually both smaller in magnitude and opposite in sign compared to the first and can often be neglected. The high energy neutron buckling is dependent on the volume and apparent reflector savings. If the rough assumption is made that the reflector savings are either small or proportional to the linear dimensions, then for equal breadth reactors of volume V the following relationship holds: $B^2 \propto V^{-2/3}$. If a reactor model half the actual volume is to be built, the buckling is increased by a factor of 1.59, requiring a reduction in the neutron age by this factor to maintain the same leakage characteristics. In water reactors, the age may be varied by changing the metal to water volume fraction. A detailed discussion of the age and cross sections is contained elsewhere (*1*). For approximate purposes the age dependence on the metal to water volume fraction v goes as $(1 + v)^m$, where m is 1, 0.87, and 0.55 for aluminum, zirconium, and steel, respectively. An age change will usually be accompanied by changes in the thermal diffusion properties.

The ratio of fuel absorptions to total absorptions in the thermal neutron energy group defines the thermal utilization. This may be written symbolically as

$$\frac{\Sigma_{\text{fuel}}\,\Phi_{\text{fuel}}}{\Sigma_{\text{average}}\,\Phi_{\text{average}}} = \text{thermal utilization}$$

where Σ and Φ are the thermal macroscopic absorption cross section and flux, respectively. The thermal utilization combined with the leakage probability determine the over-all reactivity of the core. In constructing a reduced scale reactor model, the leakage characteristics are first conserved by changing the nonfuel reactor composition to adjust the age. This determines a nonfuel absorption cross section to which fuel is added to obtain the desired thermal utilization. The amount of fuel to be added (neglecting flux disadvantage factors) is given by the following formula.

$$\Sigma_{\text{fuel}} = \left(\frac{\text{thermal utilization}}{1 - \text{thermal utilization}}\right)\Sigma_{\text{nonfuel}}.$$

The thermal neutron flux is attenuated in traversing a lumped neutron absorbing material. The ratio of the average flux within the sample to the nonattenuated flux is the disadvantage or self-shielding factor. This factor is dependent upon the size and concentration of the material and the external environment. Two neutron absorbing samples are said to be equivalent when they have the same absorption parameter ξ, which is defined as $(2v/s)\Sigma$ and is equal to the number of attenuation or relaxation lengths in the material; v is the volume, s the surface, and Σ the macroscopic cross section. The quantity $2v/s$ is the mean chord length through the sample and is divided by Σ^{-1}, the number of mean free paths between interactions. This criterion is relatively insensitive to geometry as may be seen by a comparison of the disadvantage factor for a slab, cylinder, or sphere of given ξ (*2*). Similar reactivity effects of elements will then be obtained by elements having reduced physical size and increased concentration or increased physical size and reduced concentration.

The relative strength of control rods can be accurately predicted by the absorption area method (*3*). Control strength, or neutron absorption ability, is proportional to the product of the lateral rod perimeter density and the neutron diffusion length. Control is equal to $P\Phi L/A$, where P is the lateral rod perimeter; A, the core cross sectional area; Φ, the geometric equivalence factor; and L, the neutron diffusion length. The theory of control rods for infinite slab absorbers and its conversion to other geometries has been reported (*4*), and the terminology used here is the same. The lateral rod perimeter P sums up the total linear surface area available for rod absorption. The core cross sectional area A, which is taken perpendicular to the rod axis, is a measure of the total neutron content to be affected by the rods, so that P/A is a measure of the rod density.

TABLE I

Φ FOR SIMPLE CONFIGURATIONS

Configuration	Φ
Finite slab (half width l)	$1 + (L/2l)$
Wye shape (wing width l)	$1 + (0.28L/2l)$
Cross shape (wing width l)	$1 - (0.27L/2l)$
Rectangle (side of l_1 and l_2)	$1 + [2.41L/2(l_1 + l_2)]$

The geometric equivalence factor Φ to slab rods is listed in Table I for a few simple geometries to illustrate their relationship.

Control equivalence may be had by matching the product $P\Phi L/A$ of two configurations. If in addition the ratio P/A, which is inversely proportional to the mean rod spacing, is maintained, then the rod interaction effects also will be conserved. The proper neutron diffusion length to use will depend on whether the rod is strongly thermal or epithermal.

$$L^2\,(\text{thermal}) = \frac{\text{Diffusion coefficient}}{\text{Absorption coefficient}}$$

$$L^2\,(\text{epithermal}) = \frac{\text{Diffusion coefficient}}{\text{Slowing down plus absorption cross section}}.$$

In this way it is possible to simulate control effectiveness using less control rod material in a reduced volume core of a simplified geometric form.

A description of thermal spectra may be made in terms of the ratio of Σ_a, the absorption cross section, to $\xi\Sigma_s$, the slowing down cross section just above thermal (*1*). (Here ξ is the logarithmic energy decrement per neutron collision.) The spectrum approaches a Maxwellian as the term $\Sigma_a/(\xi\Sigma_s)$ approaches zero. For example, in water-moderated reactors it is possible to approximate an apparent temperature hardening of the neutron energy distribution by an absorption hardening at room temperature by maintaining the ratio $\Sigma_a/(\xi\Sigma_s)$.

It is sometimes difficult to observe local diffusion effects such as flux peaking within prescribed geometric limits. It may be desirable in some applications to expand the geometry of the problem to accommodate the necessary instrumentation. On the other hand, if experiments are to be performed in a flat flux region of a reactor, it may be desirable to condense the configuration. The analytical solutions of diffusion phenomena can generally be expressed in terms of dimensionless parameters, i.e., ratios of the linear dimensions of the problem to the thermal diffusion length. For example, the insertion of a thermally black absorber into a cell of characteristic dimension x will be independent of cell size if the parameter x/L is maintained (neglecting flux extrapolation changes into the sample) (*4*).

The dimensionless parameters noted by no means constitute a complete list of the parameters occurring in a study of nuclear reactors. They were chosen only to illustrate that similitude principles may possibly be as useful in the field of nuclear engineering as they have been in other fields. Unless an unlimited budget is available, analytical solutions of problems, incorporating the essential features of concern, should be performed to determine what dimensionless parameters may be used to obtain experimental simplification.

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